A PARAMETRIC STUDY OF MULTI-STAGE GUN LAUNCHED ROCKETS

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SUMMARY

In order to assess the potential of multi stage gun-launched rockets, a study of the vehicle and trajectory parameters was undertaken. A digital computer program for trajectories was written and was used in an experimental manner to approach optimum performance within various sets of restricting assumptions. The approach was found to be effective and a useful orbital potential was demonstrated with reasonable design parameters.

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INTRODUCTION

During recent years interest has been growing in the use of guns to deliver payloads to high altitudes. A substantial amount of development in this area has been carried out by the Ballistic Research Laboratories of the U.S. Army and by McGill University with support from the U.S. Army through the Army Research Office and B.R.L. (Refs. 1 & 2). The latter effort has been concerned with a 16 inch gun located on Barbados (Refs. 3 & 4). The gun is smooth bored and is mounted to allow firing elevations from about 60° to near vertical. Initial developments were concerned with sub-caliber fin stabilized saboted ballistic projectiles. This class of vehicle has been fairly well proven at this stage, and operations are phasing into scientific data gathering flights. At the same time, a sub-caliber rocket assisted vehicle is being developed. (Ref. 5) This vehicle will eventually be useful for research purposes with higher performance requirements than can be achieved with pure ballistic vehicles. It is also a stage in the development of a multi-stage full bore rocket powered vehicle with orbital capabilities.

In order to provide a basis for orientation and design guidance of a program to develop such a vehicle, it was necessary to indicate achievable performance, based on suitable assumptions and corresponding design requirements.

In particular it was necessary to determine the type of trajectory which should be flown and the corresponding sizing of the rocket stages. It was decided that experimental use of a computer program for trajectory calculation was a promising approach. The intention was to sub-optimize what appeared to be the more important parameters, and then to determine the effects of other variables, with some checking to ensure that no important cross effects were neglected.

Although many computer programs for trajectory computation are available, all have certain restrictions and many include detail which would not be required in a study of this nature. It was decided that a program designed specifically for this task would provide benefits far out weighing the time and expense of writing and debugging the program, particularly since a large high speed computer (IBM 7040) with an advanced and efficient language system (Fortran IV) was available at McGill at reasonable cost. Accordingly, the program was written and a full description of it is included in this report. It has proven to be extremely useful for this study, and it can be modified with little effort for other uses, such as the production of tracking data.

The technique of experimental optimization has proven to be quite effective. Usually the combined effects of two parameters could be determined with sufficient accuracy for these studies from two computer runs comprising nine to twelve cases

(trajectories and vehicles) each. This would cost of the order of $20. in computer time and about the same for preparation and analysis. All told about 500 cases have been run to date and all of the variables considered to be of major importance have been investigated. Although it is not claimed that performance increases could not be achieved by further manipulations of the parameters, it is felt that the results to date represent a close approximation to the ultimate, such that any improvement in performance could be considered a detail refinement and not a fundamental change in design philosophy.

DESCRIPTION OF COMPUTER PROGRAM

The program calculates trajectories for multi-stage gun launched rockets. It is restricted to the three translational degrees of freedom, and to axially symmetric vehicles with constant thrust motors. The earth is assumed to be spherical, and the following functional relationships are pre-programmed: zero lift drag coefficient and normal force coefficient-angle of attack derivative vs Mach number (Fig. 1), ambient pressure and sonic speed vs altitude (Figs. 2 & 3). Internal ballistics are not calculated. Ignition of each stage occurs at a requested value of altitude, absolute flight path angle, or time after burnout of the previous stage (or launch, in the case of the first stage). The attitude of each stage during engine firing may be in the direction of the relative velocity (as for an aerodynamically stabilized vehicle), in the direction of the absolute velocity (tangent to the absolute trajectory), a fixed attitude tangent to the absolute trajectory at ignition, or a fixed attitude specified by elevation and azimuth at ignition.

The equations of motion are written with respect to axes defined as follows:

- U - along the radius vector
- V - along the horizontal velocity vector
- W - to form a right handed orthogonal triad

Position is related through direction cosines to a set of non-rotating axes defined as follows:

- X - in the equatorial plane toward the launch longitude at the time of launch
- Y - to form a right handed orthogonal triad
- Z - normal to the equatorial plane toward the north

The equations are integrated by the Kutta-Gill method (Ref. 6). The symbols used are defined in Appendix I. The mathematical relationships are listed in Appendix II, corresponding in order to the complete program listing in Appendix III.

The inputs to the program are listed in Appendix IV, along with the card formats, and the instructions for deck make-up are in Appendix V. There are twelve general inputs on the first data card for each case, followed by a card for each rocket stage containing eleven inputs. All data fields are five columns and decimal points need not be punched.

The outputs of the program are listed in Appendix VI, which also includes a sample print-out. Inputs are listed automatically at the start of each case. In addition, messages are provided at ignition, burnout, apogee and impact. Latitude and longitude are also provided at ignition, burnout and impact. Perigee and apogee heights are calculated if burnout occurs at an altitude greater than 250,000 feet and the orbit is not hyperbolic. The program continues past final stage burnout to impact only if the calculated perigee height at final stage burnout is below 50 nm. Each case is terminated by a listing of the errors in the U and W unit vectors, and the value of the UW unit vector scalar product, followed by messages concerning calculation error conditions (underflow may be neglected) and computing time.

EXPERIMENTS AND RESULTS

The work completed so far has been restricted to orbital capabilities of a three stage rocket, with variations in gun elevation, first and second stage ignition points, first and second stage thrust direction, and third stage weight (with fixed all up weight). It is obvious that specific impulse, mass fraction and muzzle velocity are major factors in determining performance, but their primary contribution is to the ideal velocity increment. In fact, specific impulse and mass fraction may be traded off against one another or against payload with comparative impunity by maintaining the ideal velocity increment of each stage at the original value. Errors would arise due to relatively slight changes in ballistic coefficient after first stage ignition, but most of the drag loss has been incurred prior to this and these errors should not be significant. Similar calculations with muzzle velocity as an additional parameter are not likely to be as accurate due to drag losses at low altitude and trajectory changes, but could be attempted as a first approximation (i.e. an increase in muzzle velocity could be interpreted as a decrease in all up weight, an increase in payload weight, or decreases in specific impulse or mass fraction, by maintaining the same total ideal velocity increment).

The general approach consisted of selecting arbitrarily an apparently reasonable set of parameters, and then varying two parameters at a time with three or four values each.

One of the variables was always gun elevation, and the apogee and perigee heights after third stage burnout were plotted against gun elevation to determine the achievable circular orbit heights and the corresponding gun elevations. Typical results are sketched in Fig. 4. Apogee and perigee heights are represented by two curves against gun elevation. Where circular orbits are possible, the curves cross, and exchange identification because of the definitions of apogee and perigee. Both curves however are continuous provided that the flight path angle at third stage burnout is zero (i.e. the velocity vector at this point is horizontal). One curve represents the height at burnout, the other represents the height half-way around the earth. Where the ignition time and thrust direction of the third stage are not adjusted correctly to produce zero flight path angle at burnout, the curves cannot quite meet and are modified in the region near the intersection such that apogee and perigee heights respectively are continuous functions of gun elevation.

The following parameters were held constant at the indicated values (latitude, longitude, azimuth, [redacted] launch weight and cross section area refer to the present 16.4 inch gun installation at Barbados).

Muzzle Height	150 ft.
Latitude	13.07 deg.
Longitude	-59.48 deg.
Azimuth	118.23 deg.
Muzzle Velocity	4500 ft. sec.

Launch Weight	2000 lbs.
Number of Stages	3
Cross Section Area	1.478 $ft.^2$
Nozzle Exit Area	0.785 $ft.^2$
Burning Time	20 sec.
Mass Fraction	0.8
Specific Impulse	300 sec.

The third stage was constrained to fire horizontally, and the absolute flight path angle at ignition was adjusted to the nearest 0.01 deg. to keep the absolute flight path angle at burnout within 0.01 deg.

The initial set of parameters included a fairly light payload of 16 lbs., geometric staging (equal ideal velocity increments), first stage ignition at 100,000 ft., and first and second stage thrust direction type 0 (see list of firing indicator values, Appendix IV). The first parameter varied in addition to gun elevation was the second stage ignition point, and the results are shown in Fig. 5. It is obvious that the second stage should be ignited as soon as possible after first stage burnout. A similar effect for first stage ignition is indicated in Fig. 6 (16 lbs. payload) and Fig. 7 (40 lbs. payload), although in the latter case the increase in drag losses due to ignition at low altitudes evidently is a limiting factor, emphasized by the low launch angle. Fig. 6 also shows checkpoints indicating the effect of second stage ignition delay and first stage firing direction. It is apparent that performance is not extremely sensitive to these factors.

The effect of second stage ignition delay was rechecked for a 40 lb. payload, and at the same time the effect of second stage firing angle was investigated (Fig. 8). Again, the effects are not of major importance within reasonable ranges of values; the desirability of early second stage firing is still evident, and it is also clear that a g-turn trajectory is close to the optimum.

Attention was then focussed on stage sizing. Since the first and second stages should be fired in quick succession, it was assumed that geometric sizing (equal velocity increments) would be near optimum for these stages, and the more significant effect would be the weight of the third stage (and thus the contribution of the third stage to the total ideal velocity increment). This effect is shown in Figs. 9, 10 & 11. Again, performance is not extremelv sensitive to parameter variations. The optimized configuration yields a slight increase in circular orbit height, and exhibits rather large variations from geometric staging (Fig. 12). Comparison of Fig.6 & Fig.7 indicates that the first stage ignition altitude should increase as the payload increases. Thus the performance indicated in Fig. 11 (constant first stage ignition altitude at 25,000 ft) is probably less than that achievable with the heavier payloads. In fact, the point at 50 lb. payload required lowering the second stage firing angle, and stage weights were not optimized since it probably

would have been necessary to consider the weights of all three stages as well as the second stage firing angle and the gun elevation. It may be possible to retain a g-turn trajectory with later ignition.

It is interesting to note that as the parameter values approach the optimum, the gun elevation approaches the neighbourhood of 45°, and the slopes of the curves of apogee and perigee height vs. gun elevation approach equal magnitude (with opposite signs). Conversely, for any condition which is far from optimum, these characteristics are not apparent.

It is of interest to examine the losses involved in the trajectories considered. Considering two payloads (16 and 40 lb.), optimized third stage weight and first stage ignition at 25,000 ft. (not optimum for 40 lb. payload) we have the following energy balances:-

Payload Weight - lb.		16	40
Input			
Muzzle Velocity	ft/sec	4500	4500
Rocket Velocity Increment	ft/sec	29019	25338
Total Velocity Increment	ft/sec	33519	29838
Energy	10^6 ft^2/sec^2	561.8	445.2
Initial State			
Radius	10^6ft	20.93	20.93
Velocity	ft/sec	1492	1492
Potential Energy	10^6 ft^2/sec^2	-672.7	-672.7
Kinetic Energy	10^6 ft^2/sec^2	1.1	1.1
Total	10^6 ft^2/sec^2	-671.6	-671.6

Final State

Radius	10^6 ft	30.18	23.11
Velocity	ft/sec	21598	24682
Potential Energy	10^6 ft^2/sec^2	-466.4	-609.2
Kinetic Energy	10^6 ft^2/sec^2	233.2	304.6
Total	10^6 ft^2/sec^2	-233.2	-304.6

Energy Loss

Absolute	10^6 ft^2/sec^2	123.4	78.2
Percent of Input		22.0	17.6

In order to show the effects of specific impulse and mass fraction, it seems reasonable to select a total ideal rocket velocity increment which is representative of that required for circular orbit entry using the 16 inch gun at Barbados. This has been done for a velocity of 24,400 ft/sec and the results are shown in Fig. 14. Geometric staging was assumed.

Typical trajectory characteristics are shown in Fig. 15 (height vs. range), Fig. 16 (height vs. airspeed) and Fig.17 (height and airspeed vs. time). Typical vehicle parameters are listed below.

STAGE	Weight -- Lb. Propellant	Remainder	Total
1	1152	288	1440
2	320	80	400
3	96	24	120
Payload	0	40	40
3 & Payload	96	64	160
2, 3 & Payload	416	144	560
1, 2, 3 & Payload	1568	432	2000

CONCLUSIONS AND RECOMMENDATIONS

The orbital capabilities of a multi-stage gun launched rocket have been demonstrated. In particular, it has been shown that with the existing 16 inch gun installation at Barbados, launching a 2000 lb. three stage vehicle at 4500 ft/sec., a payload of 40 lb. can be placed in a 400 nm circular orbit (based on a specific impulse in vacuum of 300 sec. and a propellant mass fraction of 0.8)

Optimum trajectories for entry into circular orbits are close to the g-turn type, with gun elevation in the neighborhood of 45°, first stage ignition in the 25-100,000 ft. range and second stage ignition as soon as possible after first stage burnout. Optimum vehicles involve third stage velocity increments somewhat less than one-third of the total (for three stage vehicles) but the penalty of equal velocity increment staging is small.

Further work in this area could be directed toward heavier payloads (or lower specific impulse, or lower mass fraction), lower muzzle velocity, long burning times, higher gun elevations, and variations in drag (or atmospheric properties), launch location, and azimuth.

It is hoped that the optimization procedure will be programmed in the near future, so that the optimum vehicle and/or trajectory will be produced automatically, at least for a limited number of parameters.

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FIGURE 1

AERODYNAMIC CHARACTERISTICS

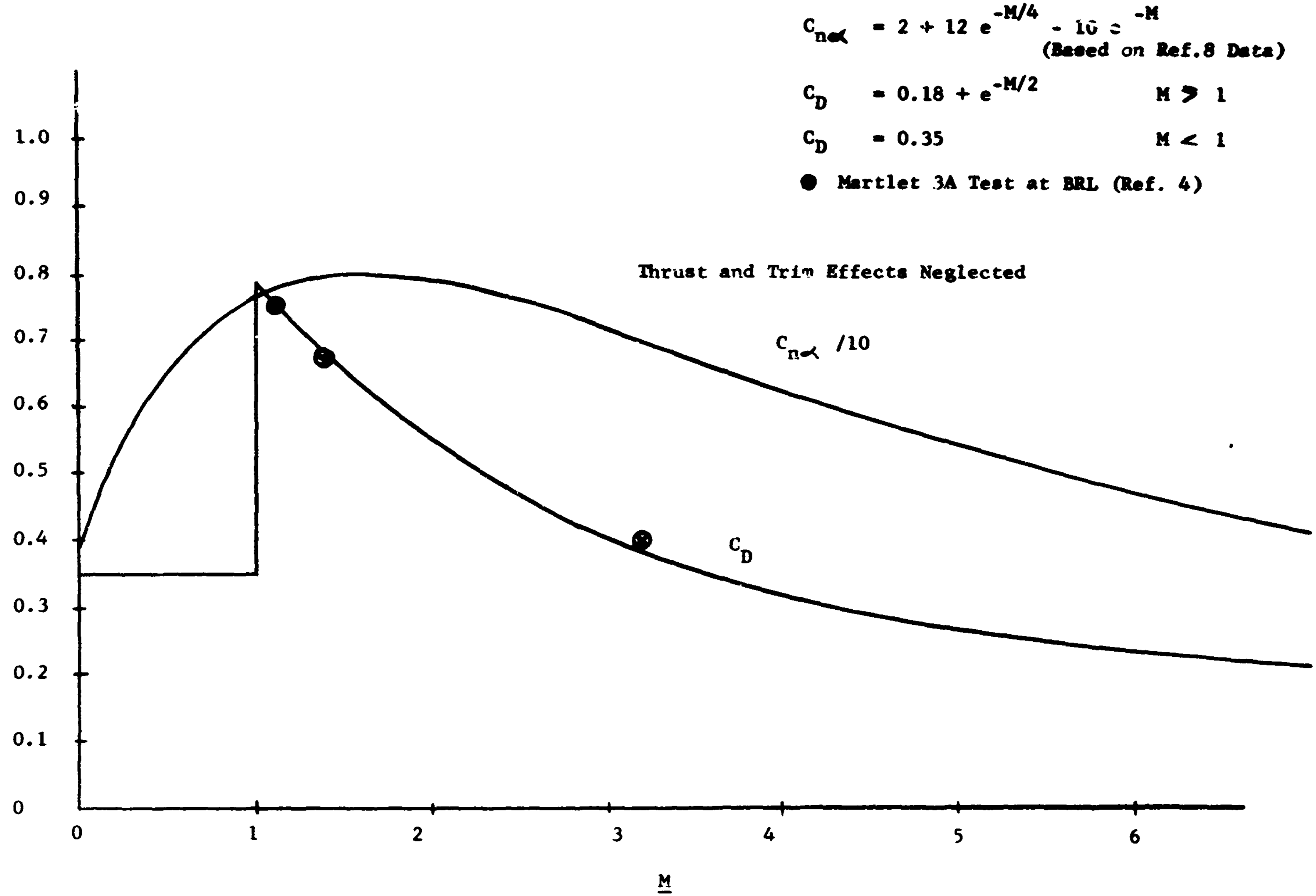


FIGURE 2

ATMOSPHERIC PRESSURE APPROXIMATION ERROR

Based on U.S. 1962 Standard Atmosphere (Ref. 5)

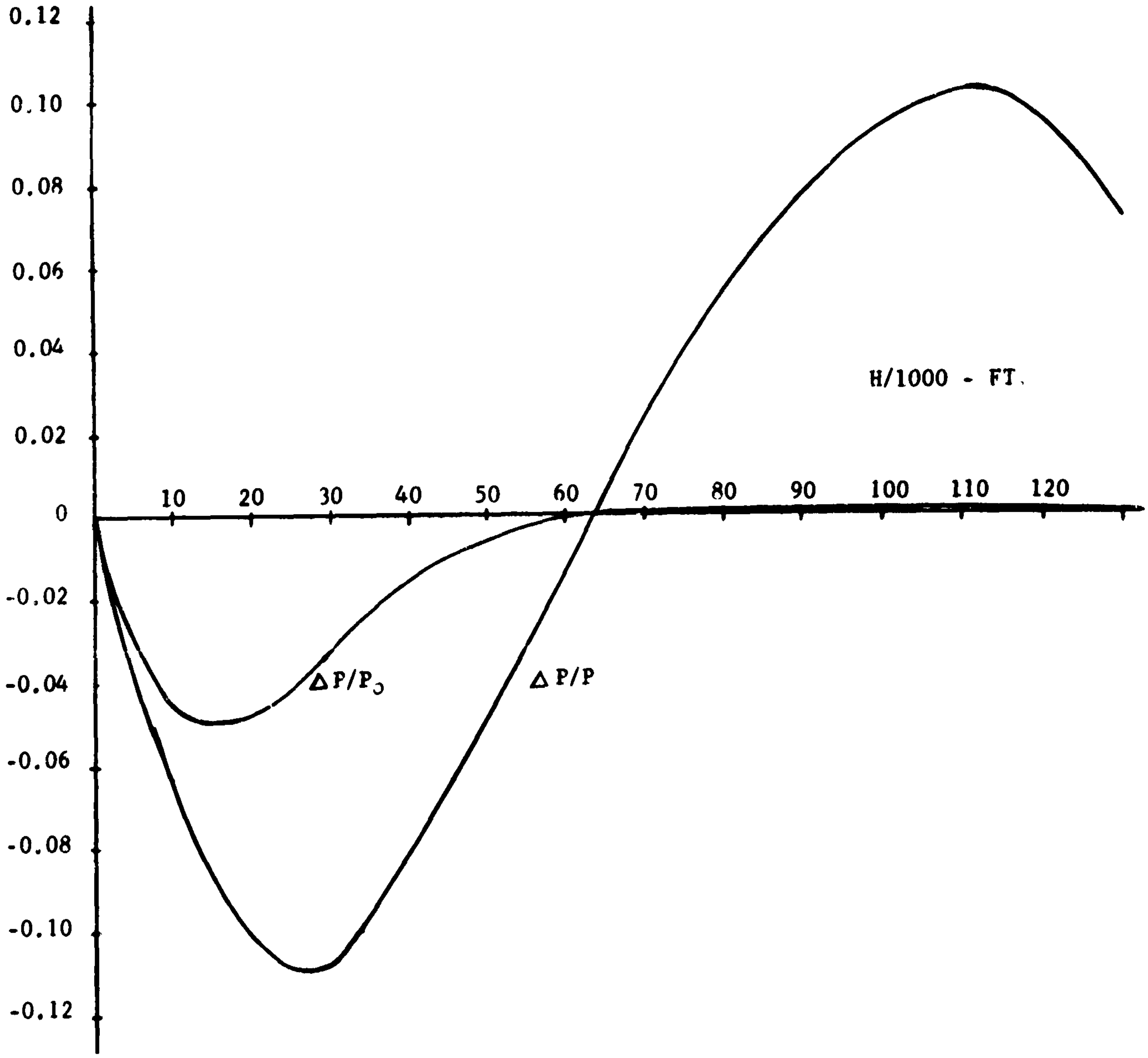


FIGURE 3

SONIC SPEED APPROXIMATION ERROR

Based on U.S. 1962 Standard Atmosphere (Ref. 5)

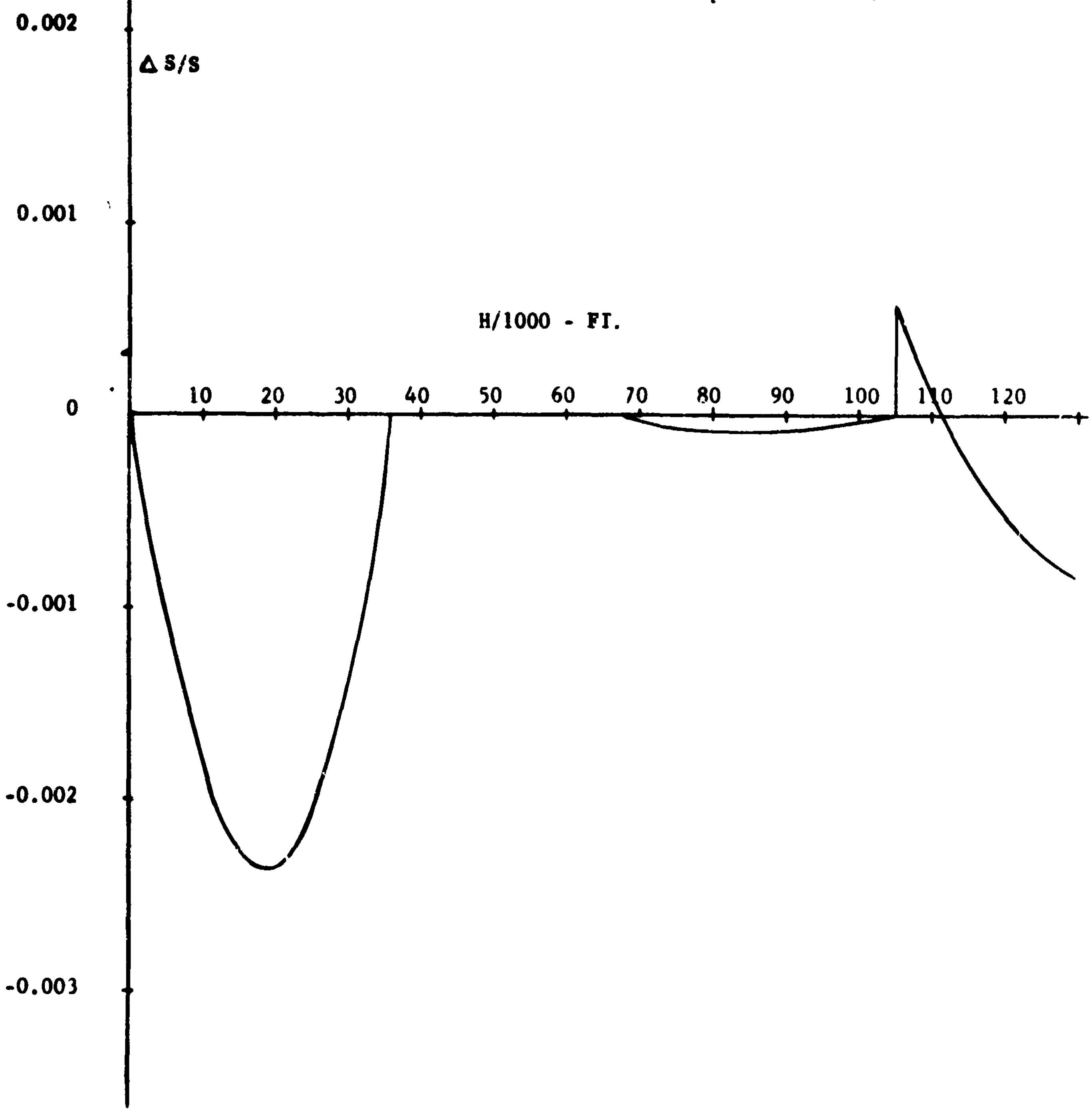
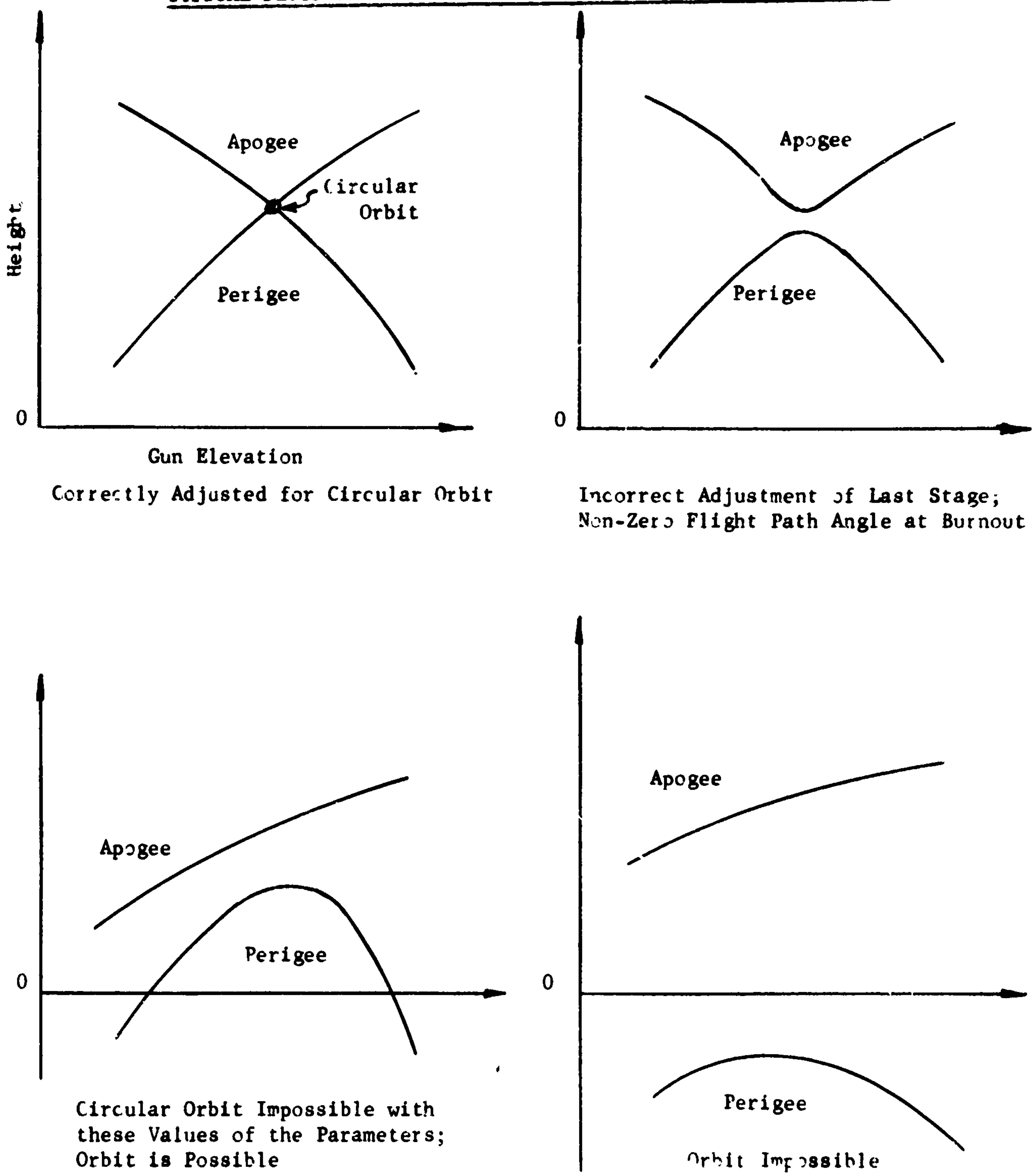


FIGURE 4

TYPICAL PLOTS OF APOGEE AND PERIGEE HEIGHT VS. GUN ELEVATION



Correctly Adjusted for Circular Orbit

Incorrect Adjustment of Last Stage; Non-Zero Flight Path Angle at Burnout

Circular Orbit Impossible with these Values of the Parameters; Orbit is Possible

Orbit Impossible

FIGURE 5

EFFECT OF SECOND STAGE IGNITION POINT

Payload 16 Lb.
Third Stage Weight 64 Lb. (Geometric Staging)
First Stage Ignition at 100,000 Ft.
First & Second Stage Thrust Direction Type 0

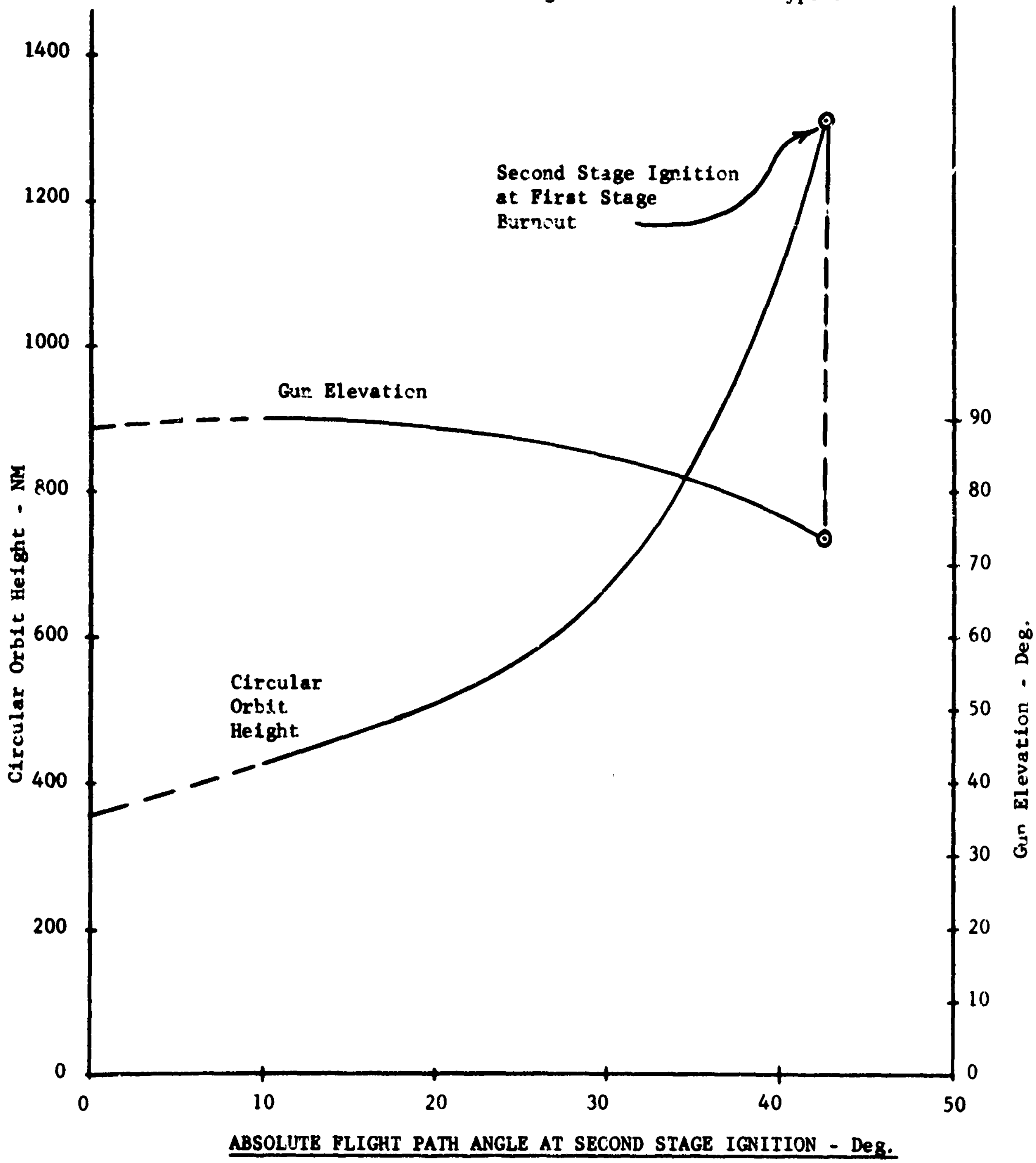
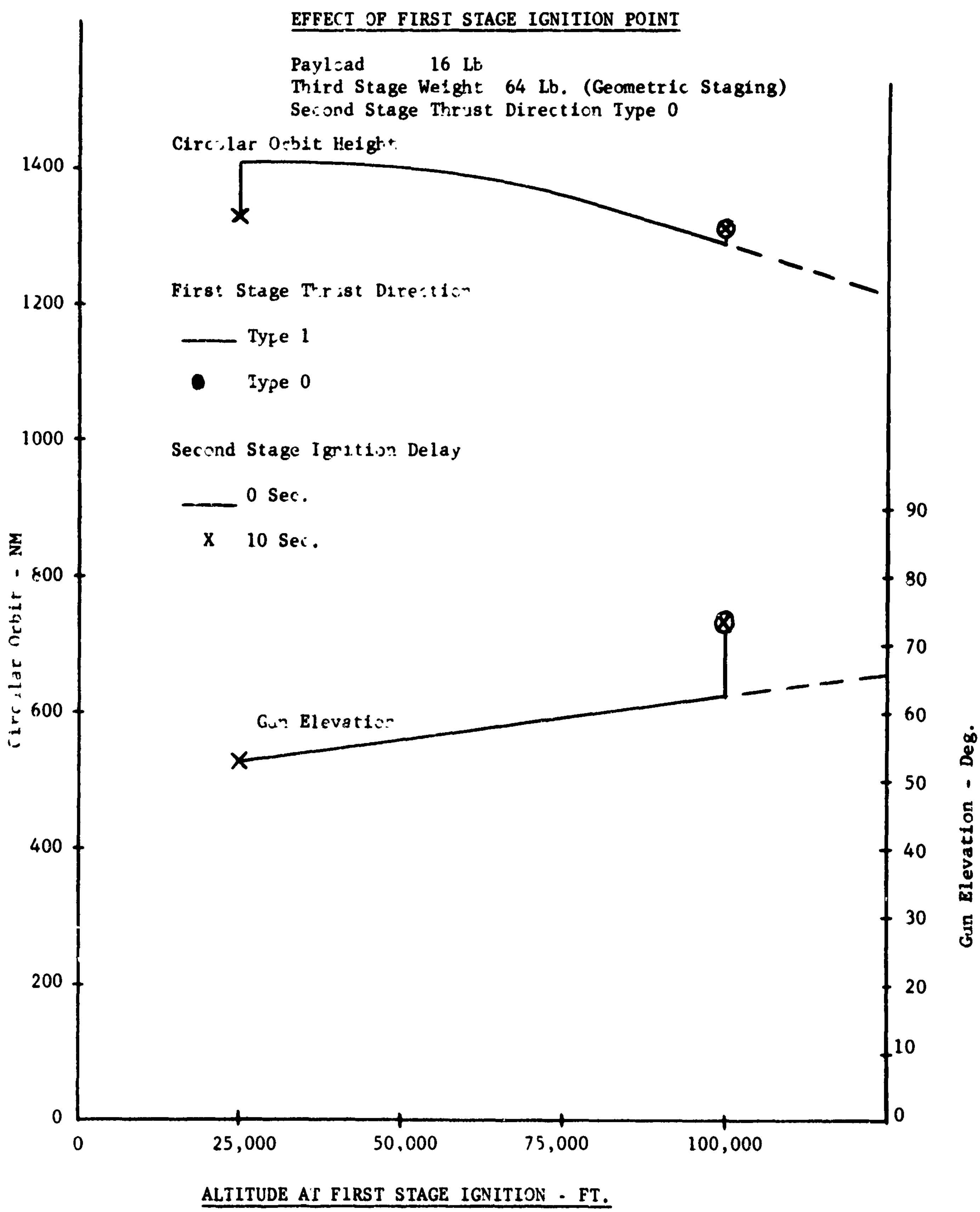


FIGURE 6



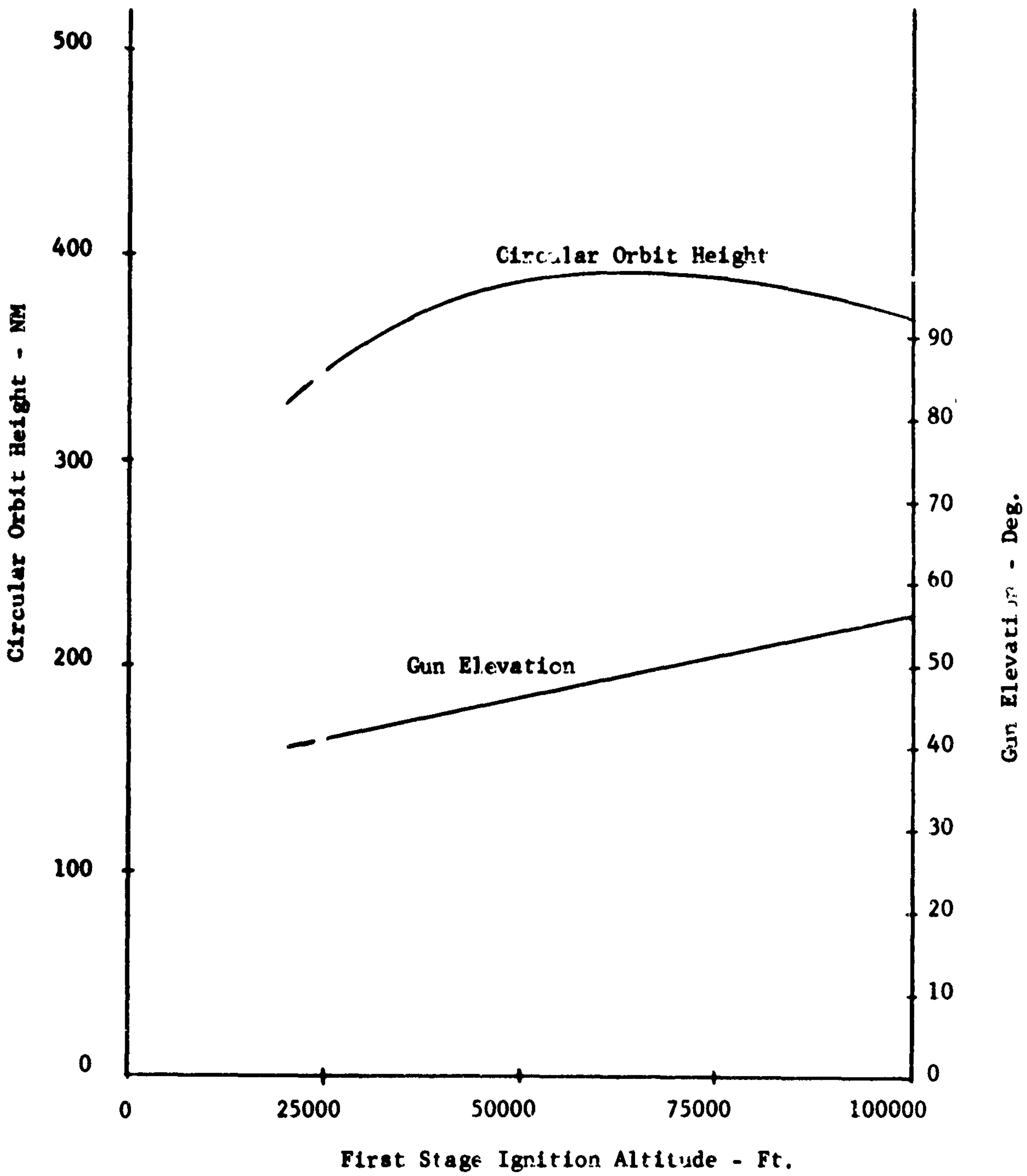
EFFECT OF FIRST STAGE IGNITION POINT
Payload 16 Lb
Third Stage Weight 64 Lb. (Geometric Staging)
Second Stage Thrust Direction Type 0
Circular Orbit Height
First Stage Thrust Direction
Type 1
Type 0
Second Stage Ignition Delay
0 Sec.
X 10 Sec.
Gun Elevation
Circular Orbit - NM
1400
1200
1000
800
600
400
200
0
Gun Elevation - Deg.
90
80
70
60
50
40
30
20
10
0
0
25,000
50,000
75,000
100,000
ALTITUDE AT FIRST STAGE IGNITION - FT.

FIGURE 7

EFFECT OF FIRST STAGE IGNITION POINT

Payload 40 Lb.
Third Stage Weight 140 Lb.

Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0



F I G U R E 8

EFFECT OF SECOND STAGE IGNITION DELAY

AND THRUST DIRECTION

Payload 40 Lb.
Third Stage Weight 140 Lb.

First Stage Ignition at 25000 Ft.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 2

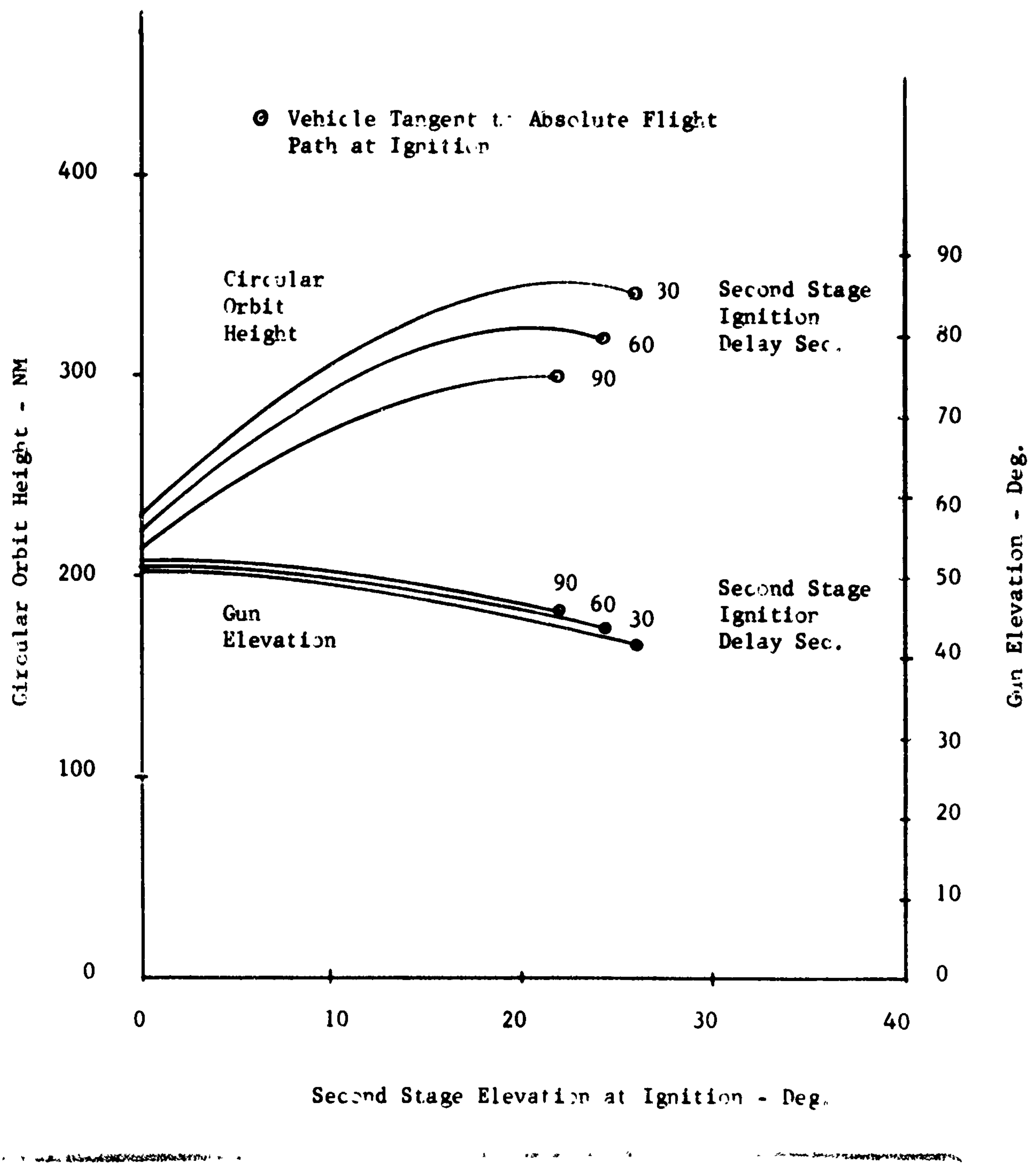
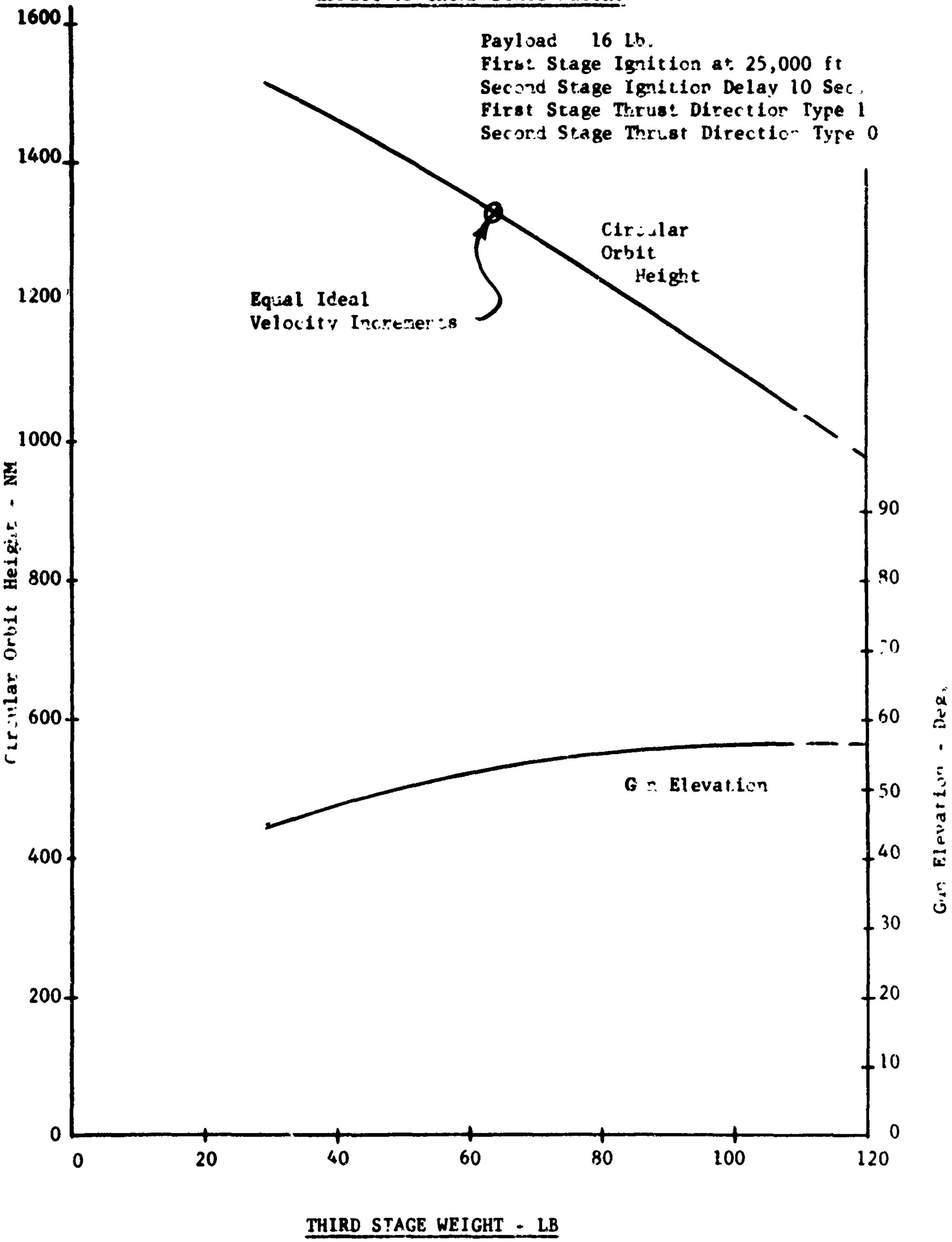


FIGURE 9

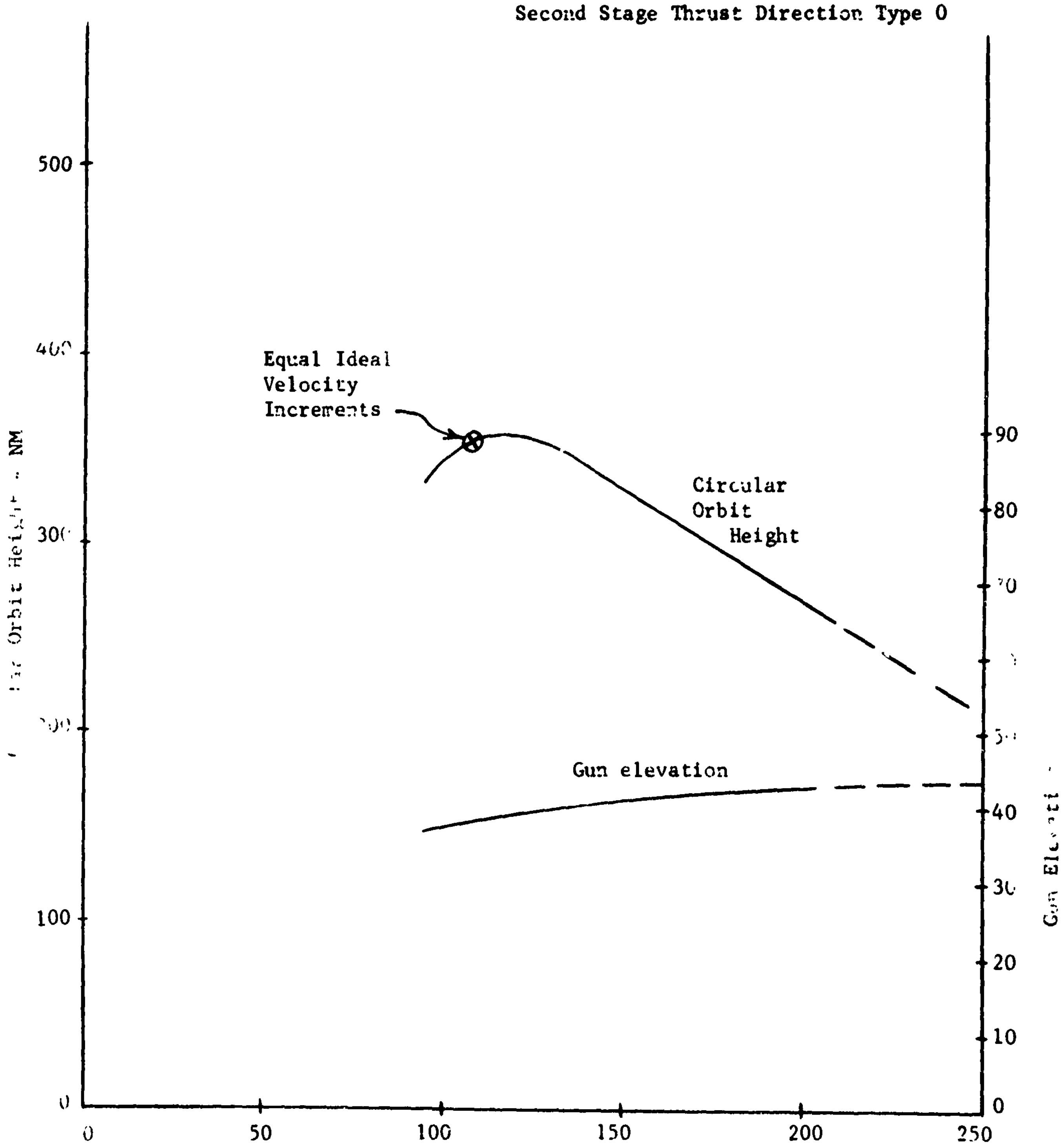
EFFECT OF THIRD STAGE WEIGHT



F I G U R E 10

EFFECT OF THIRD STAGE WEIGHT

Payload 40 Lb.
First Stage Ignition at 25,000 ft.
Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0



THIRD STAGE WEIGHT - LB

F I G U R E 11

EFFECT OF OPTIMIZING THIRD STAGE WEIGHT

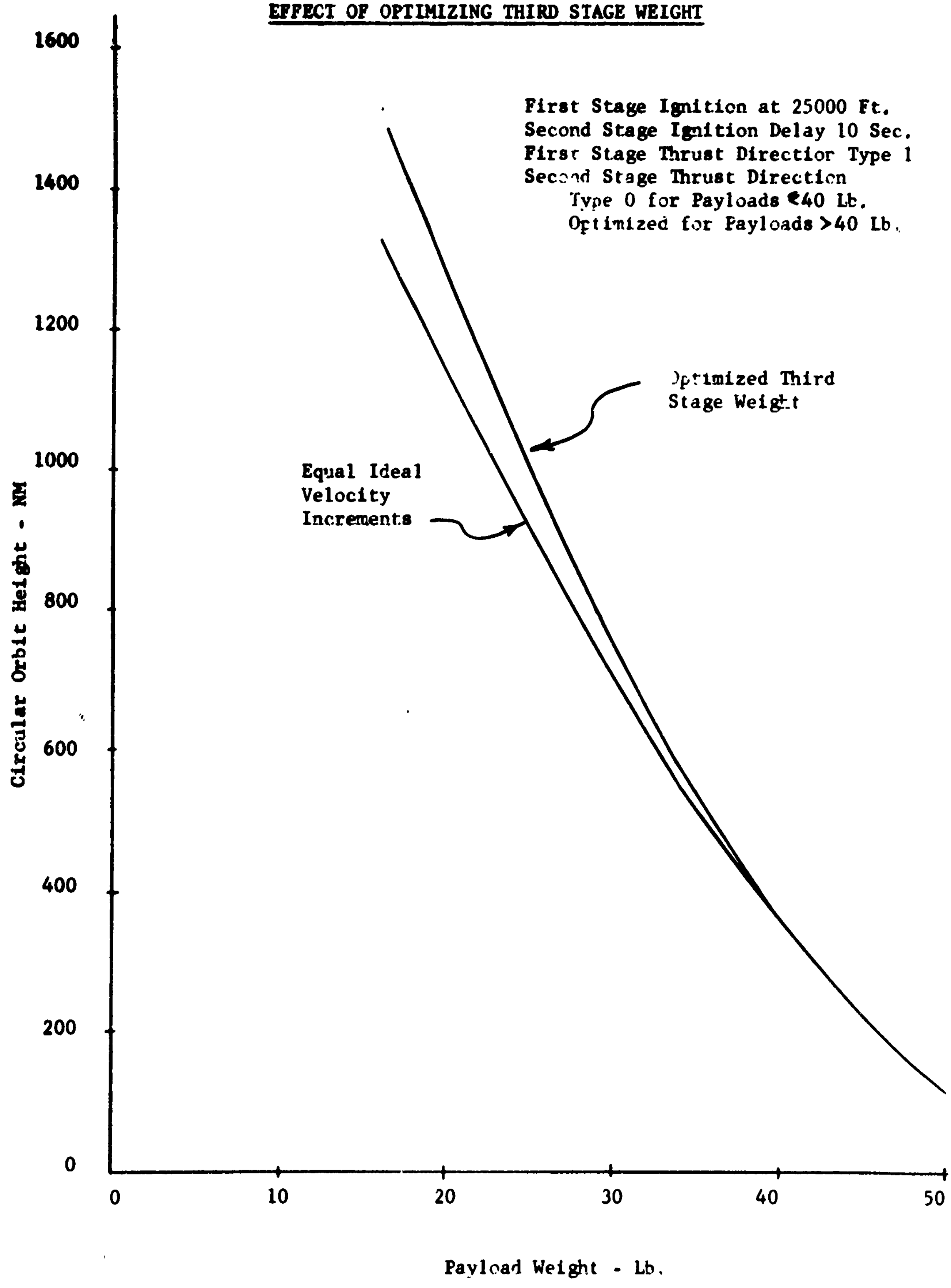
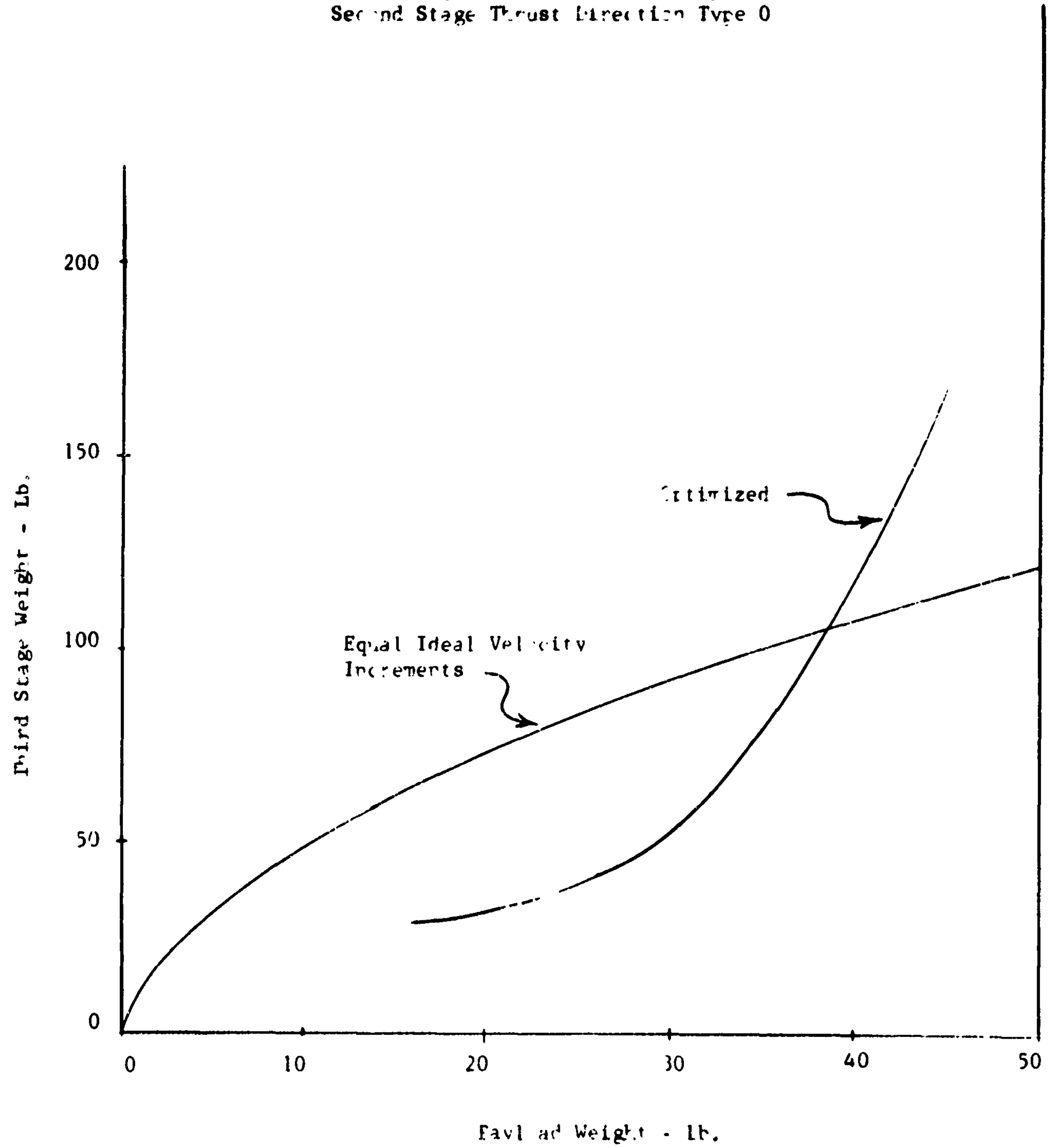


FIGURE 12

COMPARISON OF NOMINAL AND OPTIMIZED

THIRD STAGE WEIGHT

First Stage Ignition at 25000 Ft.
Second Stage Ignition Delay 10 Sec.
First Stage Thrust Direction Type 1
Second Stage Thrust Direction Type 0



F I G U R E 13

EFFECT OF SECOND STAGE THRUST DIRECTION

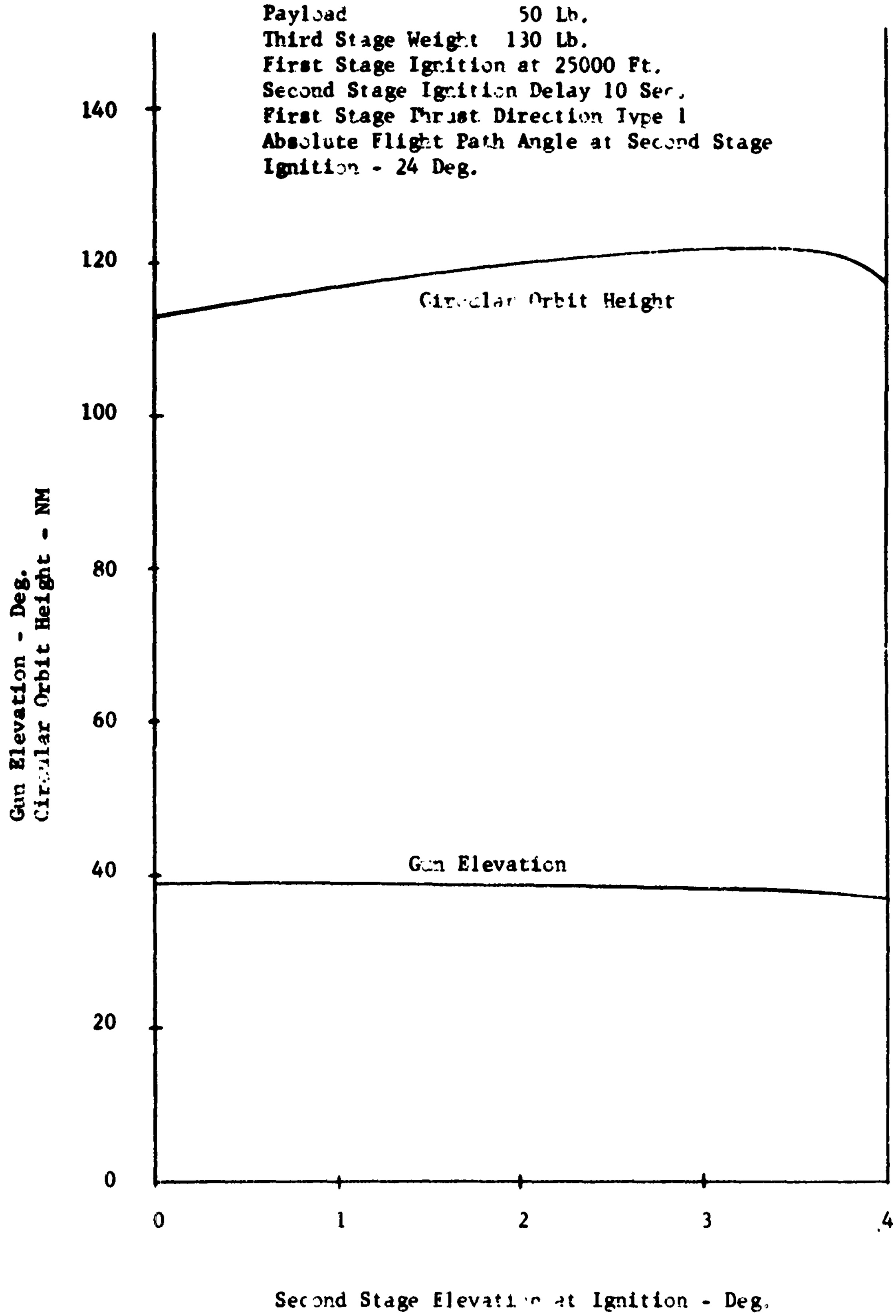
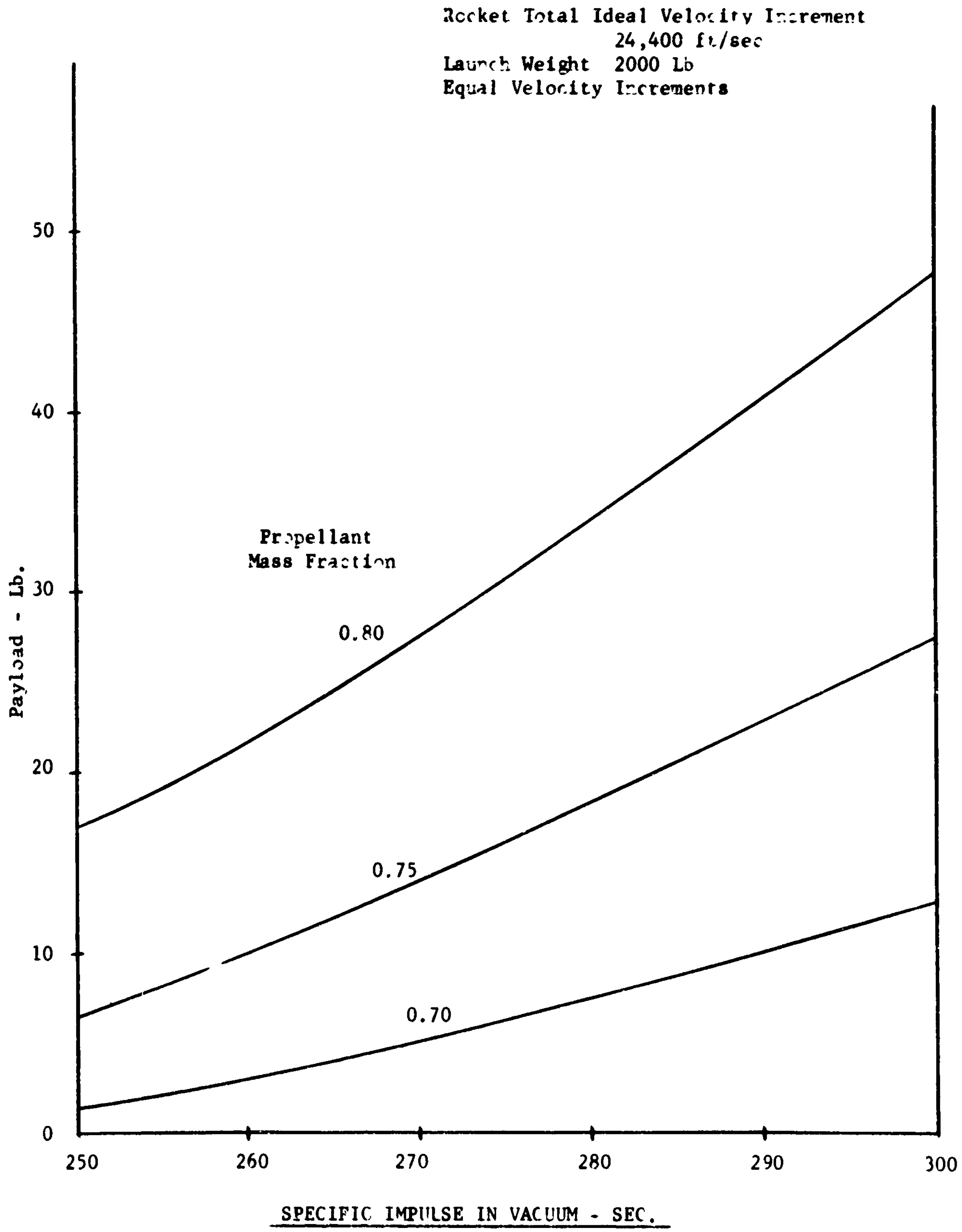


FIGURE 14

EFFECT OF SPECIFIC IMPULSE AND MASS FRACTION



F I G U R E 15

TYPICAL TRAJECTORY - HEIGHT VS. RANGE

Payload 40 Lb.
Specific Impulse 300 Sec.
Mass Fraction 0.8
Burning Times 20 Sec.
Gun Elevation 39 Deg.
First Stage Ignition at 25,000 Ft.
First Stage Direction Type 1
Second Stage Ignition Delay 10 Sec.
Second Stage Direction Type 0

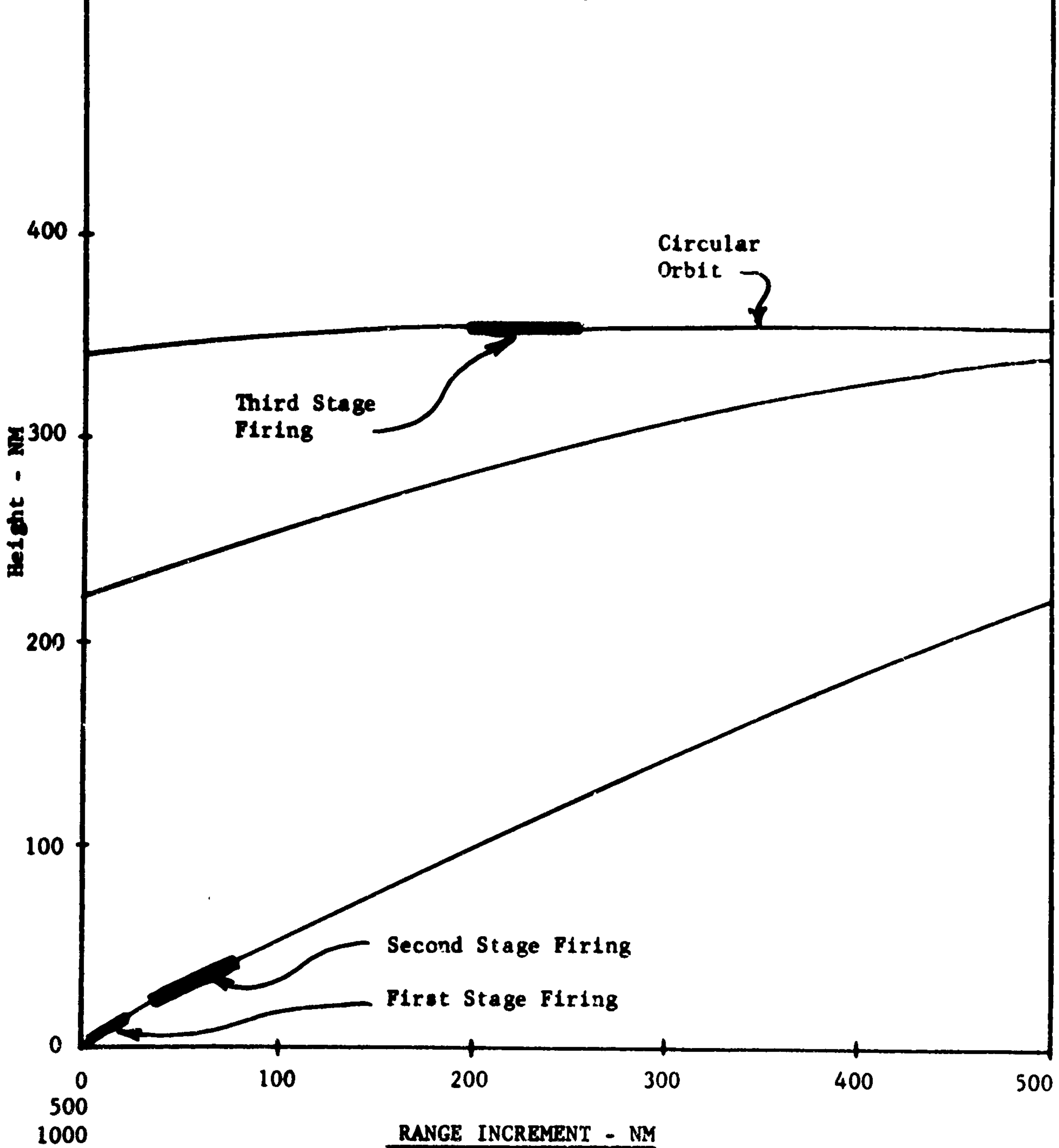
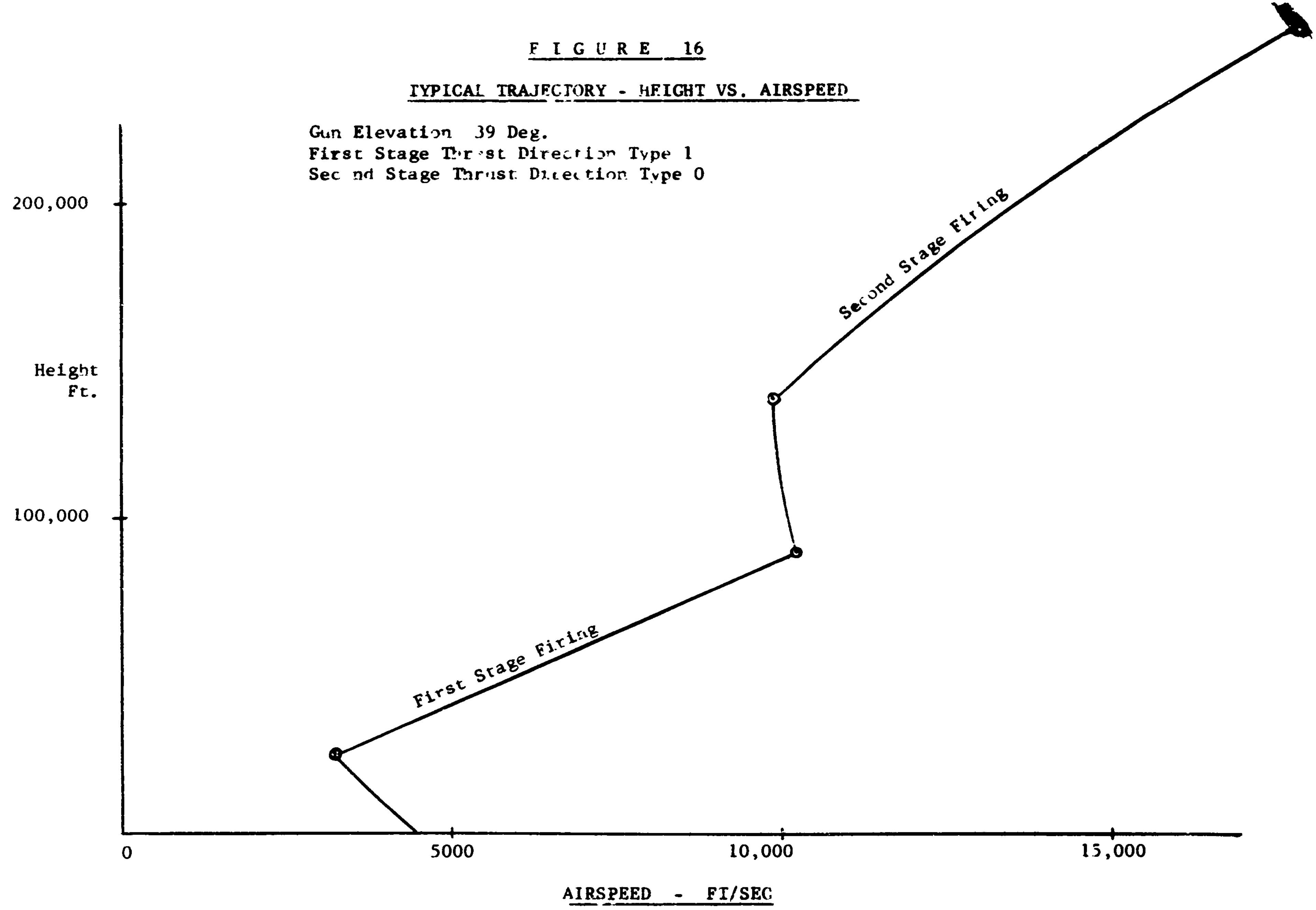


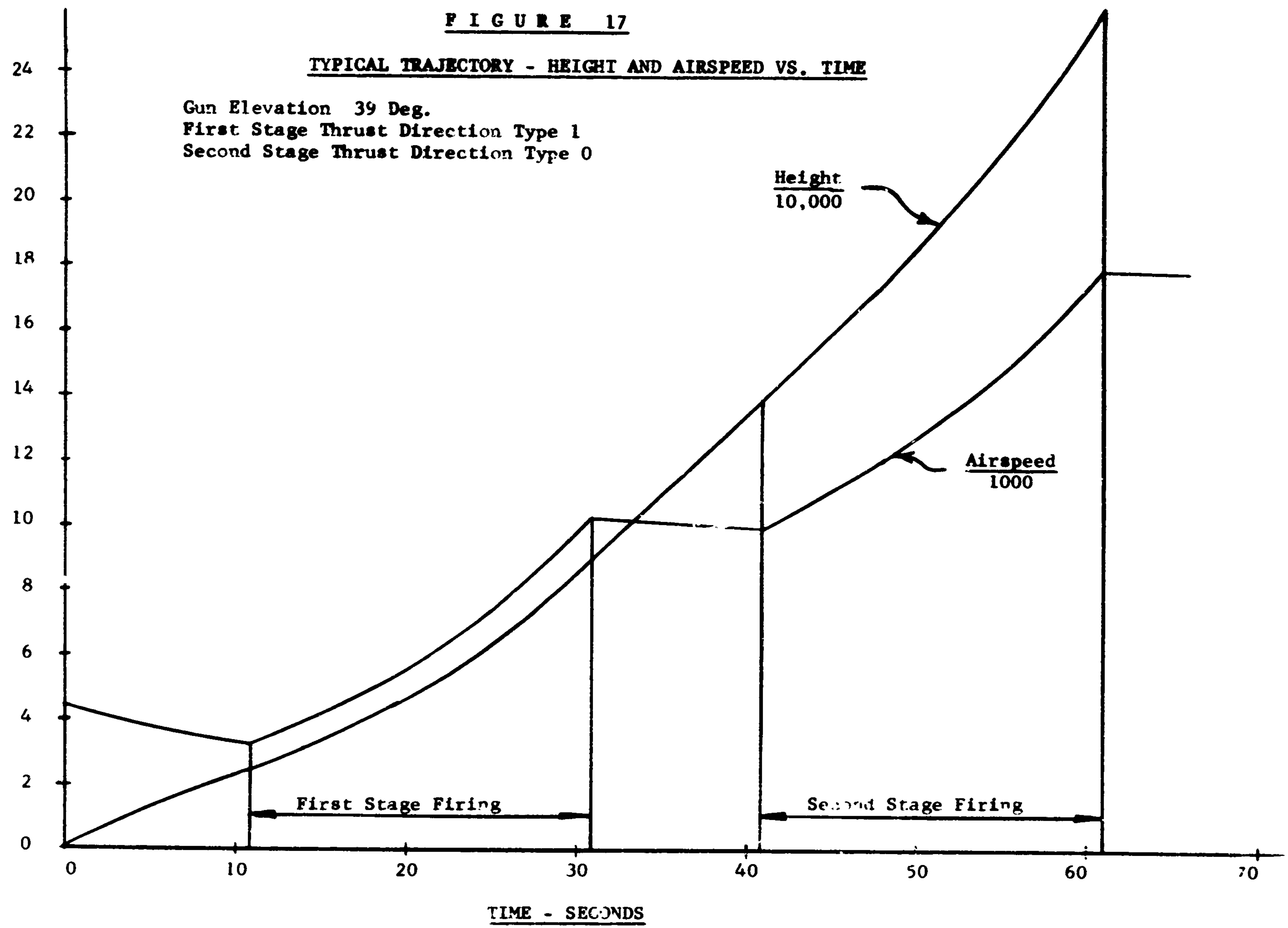
FIGURE 16

TYPICAL TRAJECTORY - HEIGHT VS. AIRSPEED



F I G U R E 17

TYPICAL TRAJECTORY - HEIGHT AND AIRSPEED VS. TIME



APPENDIX I

LIST OF SYMBOLS

FORTRAN	MATH	DEFINITION
AP	P	Atmospheric Pressure
AT	ϕ	Latitude (See Note (2))
ATD	ϕ^{o}	Latitude (Degrees)
ATG	ϕ_{g}^{o}	Latitude of Gun (Degrees)
ATO	ϕ_{g}	Latitude of Gun
AVU		Length of Unit Vector on U Axis Minus 1.
AVW		Length of Unit Vector on W Axis Minus 1.
AX	a	Semi-Major Axis of Orbit
AZ	β	Azimuth (See Note (1))
AZD	β^{o}	Azimuth (Degrees)
AZG	β_{g}^{o}	Azimuth of Gun (Degrees)
AZS (N)	β_{s}	Azimuth of Thrust Vector at Ignition (Degrees)
C (I)	K_{i}	Kutta-Gill K for Variable I
CAZ	$\cos\beta$	
CD	C_{d}	Drag Coefficient
CEL	$\cos\delta$	
CLA	$\cos\phi$	
CNA	$C_{n_{\alpha}}$	Normal Force Coefficient-Angle of Attack Derivative
CT	C	Iteration Count
DL		Computing Time in 60ths of a Second
DLS		Computing Time in Seconds
DR	D	Drag

FORTRAN	MATH	DEFINITION
DT	Δt	Step Size
DTB (N)	Δt_b	Burning Time
DTD	Δt_p	Possible Step Size; Previous Step Size
DTI	Δt_I	Nominal Step Size
DTP	Δt_p	Nominal Print-Out Interval
DU	δ_u	U-Direction Cosine of Thrust Vector (Body Axis)
DV	δ_v	V-Direction Cosine of Thrust Vector (Body Axis)
DW	δ_w	W-Direction Cosine of Thrust Vector (Body Axis)
DX	δ_x	X-Direction Cosine of Thrust Vector (Body Axis)
DY	δ_y	Y-Direction Cosine of Thrust Vector (Body Axis)
DZ	δ_z	Z-Direction Cosine of Thrust Vector (Body Axis)
EA (N)	A	Exhaust Area
EL	δ	Elevation
ELD	δ°	Elevation (Degrees)
ELG	δ_g°	Elevation of Gun (Degrees)
ELS (N)	δ_s°	Elevation of Thrust Vector at Ignition (Degrees)
MM	M	Mach Number
F	F	$108/\pi$
FN	F_n	Normal Force
FNU	F_{nu}	U-Component of Normal Force
FNV	F_{nv}	V-Component of Normal Force
FNW	F_{nw}	W-Component of Normal Force

FORTRAN	MATH	DEFINITION
FR (N)	f	Mass Fraction
GC	k_e^2	Gravitational Constant
GO	g_o	Acceleration due to Gravity at S.L.
HA	H_A	Apogee Height
HP	H_p	Perigee Height
HT	H	Height (Feet)
HTN		Height (Nautical Miles)
I	i	Variable Number
I1 (N)	I_1	Ignition Indicator
I2 (N)	I_2	Firing Indicator (Direction)
J	N	Integration Step
K		Error Indicator
L		Computer Clock Reading at End of Run
LO		Computer Clock Reading at Beginning of Run
M		Case Number
N	n	Stage Number
NL	n_L	Number of Lines Printed this Page
NN	n^1	Previous Stage Number
NP	n_p	Page Number
NS	n_s	Number of Stages
ØN	θ	Longitude (See Note (4))
ØND	θ^o	Longitude (Degrees)
ØNG	θ_g^o	Longitude of Gun (Degrees)
ØNO	θ_g	Longitude of Gun
PA	γ	Absolute Flight Path Angle (See Note (2))

FORTRAN	MATH	DEFINITION
PAD	γ°	Absolute Flight Path Angle (Degrees)
PAP	γ'	Absolute Flight Path Angle (Previous Step)
Q (I)	Q_i	Kutta-Gill Q for Variable I
QN		
QS	QS	
QT (N)	Q_n	Ignition Value (See Note (5))
RA	d	Range (Feet)
RAN		Range (Nautical Miles)
RE	r_e	Earth Radius
RP	r_p	Perigee Radius
SA (N)	S	Cross Section Area
SAZ	$\sin \beta$	
SEL	$\sin \delta$	
SI (N)	I_{∞}	Specific Impulse in Vacuum
SL	p	Semi-Latus Rectum of Orbit
SLA	$\sin \phi$	
SP	Ω	Angular Rate of Earth
SPWU		Scalar Product of Unit Vectors on U and W Axes
SS	c	Speed of Sound
S3	S_3	Event Indicator
S4	S_4	Firing Indicator (Condition)
T	t	Time
TB	t_B	Time of Burnout
TH	T	Thrust
THO	T_{∞}	Thrust in Vacuum

FORTRAN	MATH	DEFINITION
TI	t_I	Time of Ignition
TØ	t_o	Time of Last Print-Out
V	V	Absolute Velocity
VR	V_r	Relative Velocity (Referred to Earth)
VRU	V_{ru}	U-Component of Relative Velocity
VRV	V_{rv}	V-Component of Relative Velocity
VRW	V_{rw}	W-Component of Relative Velocity
VX	V_x	X-Component of Unit Vector on V Axis
VY	V_y	Y-Component of Unit Vector on V Axis
VZ	V_z	Z-Component of Unit Vector on V Axis
WAZD	β_w	Azimuth of Relative Velocity
WELD	δ_w	Elevation of Relative Velocity
WI (N)	W	Total Weight Before Ignition
WU	ω_u	U-Component of Angular Velocity Vector
Y (I)	Yi	Variable I
YD (I)	$\dot{Y}i$	Time Derivative of Variable I

NOTES:

1. Azimuth as measured from the gun (the input gun direction) is measured in a horizontal plane, clockwise from North as viewed from above (farther from the earth center).

   Azimuth as measured from the vehicle (the input direction of the thrust vector at ignition and the output body axis and wind directions) is measured in a horizontal plane, clockwise from the horizontal absolute velocity vector as viewed from above.

2. Elevation and flight path angle are measured from the horizontal plane, positive upwards.

NOTES (Cont'd)

3. Latitude is measured positive North from the equator.

4. Longitude is measured positive East from Greenwich.

5. The ignition value is interpreted as
Height in thousands of feet if $I_1 = 0$

Flight path angle in degrees if $I_1 = 1$

Time after previous burnout if $I_1 = 2$

BASIC VARIABLES

VARIABLE NO. I	MATH SYMBOL	DEFINITION
1	W	Weight
2	q	Radius Rate
3	r	Radius
4	ω_w	W-Component of Angular Velocity
5	U_x	X-Component of U-Direction
6	U_y	Y-Component of U-Direction
7	U_z	Z-Component of U-Direction
8	W_x	X-Component of W-Direction
9	W_y	Y-Component of W-Direction
10	W_z	Z-Component of W-Direction

Variable Y (I)

Rate of Change of Variable YD (I)

LIST OF SYMBOLS (Cont'd)

INDICATORS

INDICATOR	VALUE	MEANING
I1	0	Ignition at Preset Height
	1	Ignition at Preset Flight Path Angle
	2	Ignition at Preset Time After Burnout*
	NOT 0,1,2	Ignition at Burnout*
I2	NOT 1,2,3	Firing at Constant Attitude Initially Tangent to Trajectory
	1	Firing in Wind Direction
	2	Firing at Preset Attitude
	3	Firing Along Trajectory
S3	-1.	Impact
	0	Burnout
	1.	Ignition or Launch or Apogee
	2.	No Event
S4	-1.	Not Firing
	0	No More Stages to Fire
	1.	Firing

* Burnout refers to the previous stage, or in the case of first stage ignition, the muzzle.

APPENDIX II

MATHEMATICS

Initialize Run

$$q = V_R \sin \delta$$

$$r = r_e + H$$

$$\omega_w = \left[(V_R \cos \delta \sin \beta + \Omega r \cos \phi)^2 + (V_R \cos \delta \cos \beta)^2 \right]^{\frac{1}{2}} / r$$

$$U_x = \cos \phi$$

$$U_y = 0$$

$$U_z = \sin \phi$$

$$W_x = -(V_R \cos \delta \sin \beta + \Omega r \cos \phi) \sin \phi / r \omega_w$$

$$W_y = -V_R \cos \delta \cos \beta / r \omega_w$$

$$W_z = (V_R \cos \delta \sin \beta + \Omega r \cos \phi) \cos \phi / r \omega_w$$

Initialize Step

$$V_x = U_z W_y - U_y W_z$$

$$V_y = U_x W_z - U_z W_x$$

$$V_z = U_y W_x - U_x W_y$$

$$V_{RU} = q$$

$$V_{RV} = r(\omega_w - \Omega W_z)$$

$$V_{RW} = r \Omega V_z$$

$$V_R = (V_{RU}^2 + V_{RV}^2 + V_{RW}^2)^{\frac{1}{2}}$$

$$H = r - r_e$$

$$V = \left[q^2 + (r \omega_w)^2 \right]^{\frac{1}{2}}$$

$$\gamma = \tan^{-1} (q / r \omega_w)$$

$$\Delta t^1 = (Q/F - \gamma) \Delta t / (\gamma - \gamma^1) \quad t \neq t_B \quad n < n_s \quad I_1 = 1$$

$$= (1000Q - H)/q \quad n < n_s \quad I_1 = 0$$

$$= Q - t + t_B \quad n < n_s \quad I_1 = 2$$

$$= -H/q \quad n = n_s \quad q < 0$$

$$= -q/\dot{q} \quad t \neq t_B \quad n = n_s \quad q > 0$$

$$\left.\begin{aligned} \delta_u &= q/V \\ \delta_v &= r\,\omega_w/V \\ \delta_w &= 0 \end{aligned}\right) \quad I_2 \neq 1,\ 2 \text{ or } 3$$

$$\left.\begin{aligned} \delta_u &= \sin \delta \\ \delta_v &= \cos \delta \cos \beta \\ \delta_w &= -\cos \delta \sin \beta \end{aligned}\right) \quad I_2 = 2$$

$$\left.\begin{aligned} \delta_x &= \delta_u U_x + \delta_v V_x + \delta_w W_x \\ \delta_y &= \delta_u U_y + \delta_v V_y + \delta_w W_y \\ \delta_z &= \delta_u U_z + \delta_v V_z + \delta_w W_z \end{aligned}\right) \quad I_2 = 1 \text{ or } 3$$

$$\dot{W} = f\ (W_{n+1} - W_n)\ /\Delta t_B$$

$$T_o = -I\dot{W}$$

$$\theta = \tan^{-1}\ (U_y/U_x) - \Omega t + \theta_o$$

$$\phi = \tan^{-1}\left[U_z/(U_x^{\ 2} + U_y^{\ 2})^{\frac{1}{2}}\right]$$

<u>Calculate Variables</u>

$$V_x = U_z W_y - U_y W_z$$

$$V_y = U_x W_z - U_z W_x$$

$$V_z = U_y W_x - U_x W_y$$

[illegible] $= q$

$$V_{RV} = r\ (\omega_w - \Omega\, W_z)$$

$$V_{RW} = r\,\Omega\, V_z$$

$$V_R = (V_{RU}^{\ 2} + V_{RV}^{\ 2} + V_{RW}^{\ 2})^{\frac{1}{2}}$$

$$H = r - r_e$$

$$P = 2116.22e^{-4.42 \times 10^{-5} H} \qquad H < 10^6$$

$$P = 0 \qquad H > 10^6$$

Calculate Thrust

$$T = T_o - AP$$

$$\left.\begin{aligned} \delta_u &= \delta_x U_x + \delta_y U_y + \delta_z U_z \\ \delta_v &= \delta_x V_x + \delta_y V_y + \delta_z V_z \\ \delta_w &= \delta_x W_x + \delta_y W_y + \delta_z W_z \end{aligned}\right\} \quad I_2 \neq 1 \text{ or } 3$$

$$\left.\begin{aligned} \delta_u &= V_{RU}/V_R \\ \delta_v &= V_{RV}/V_R \\ \delta_w &= V_{RW}/V_R \end{aligned}\right\} \quad I_2 = 1$$

$$\left.\begin{aligned} \delta_u &= q/V \\ \delta_v &= r\omega_w/V \\ \delta_w &= 0 \end{aligned}\right\} \quad I_2 = 3$$

Calculate Drag and Normal Force

$$M = V_R/c$$

$$C_d = 0.35 \qquad M < 1$$

$$C_d = 0.18 + e^{-M/2} \qquad M > 1$$

$$QS = 0.7\ PM^2\ S$$

$$D = C_d\ QS$$

$$C_{n\alpha} = 2 + 12\ e^{-M/4} - 10\ e^{-M}$$

$$Q_n = C_{n\alpha}\ QS/V_R$$

$$F_{nu} = Q_n \left[\delta_u\ (\delta_v V_{RV} + \delta_w V_{RW}) - V_{RU}\ (\delta_v^2 + \delta_w^2)\right]$$

$$F_{nv} = Q_n \left[\delta_v\ (\delta_u\ V_{RU} + \delta_w V_{RW}) - V_{RV}\ (\delta_u^2 + \delta_w^2)\right]$$

$$F_{nw} = Q_n \left[\delta_w\ (\delta_u\ V_{RU} + \delta_v V_{RV}) - V_{RW}\ (\delta_u^2 + \delta_v^2)\right]$$

$$F_n = (F_{nu}^2 + F_{nv}^2 + F_{nw}^2)^{\frac{1}{2}}$$

Calculate Rates

$$\dot{q} = r\omega_w^2 + (g_o/W)(T\delta_u - D\,V_{RU}/V_R + F_{nu}) - k_e^2/r^2$$

$$\dot{r} = q$$

$$\dot{\omega}_w = \left[-2\,q\omega_w + (g_o/W)(T\delta_v - D\,V_{RV}/V_R + F_{nv})\right]/r$$

$$\omega_u = (T\delta_w - D\,V_{RW}/V_R + F_{nw})\; g_o/Wr\,\omega_w$$

$$\dot{U}_x = V_x\,\omega_w$$

$$\dot{U}_y = V_y\,\omega_w$$

$$\dot{U}_z = V_z\,\omega_w$$

$$\dot{W}_x = -V_x\,\omega_u$$

$$\dot{W}_y = -V_y\,\omega_u$$

$$\dot{W}_z = -V_z\,\omega_u$$

Output

$$\theta = \tan^{-1}(U_y/U_x) - \Omega t + \theta_o$$

$$\phi = \tan^{-1}\left(U_z/\sqrt{U_x^2 + U_y^2}\right)$$

$$\beta = \tan^{-1}\left[\sin(\theta - \theta_o)\cos(\phi - \phi_o)/\sin(\phi - \phi_o)\right]$$

$$d = r_e \tan^{-1}\left[|\sin(\theta - \theta_o)| \,/\, |\sin\beta| \cos(\theta - \theta_o)\right]$$

$$\delta_w = \tan^{-1}\left(V_{RU}/\sqrt{V_{RV}^2 + V_{RW}^2}\right)$$

$$\beta_w = \tan^{-1}(-V_{RW}/V_{RV})$$

$$\left.\begin{array}{l} \delta_B = \delta_w \\ \beta_B = \beta_w \end{array}\right) \quad \text{Not firing}$$

$$\left.\begin{array}{l} \delta_B = \tan^{-1}\left(\delta_u/\sqrt{\delta_v^2 + \delta_w^2}\right) \\ \beta_B = \tan^{-1}(-\delta_w/\delta_v) \end{array}\right) \quad \text{Firing}$$

$$a = 1/(2/r - V^2/k_e^2)$$

$$p = (Vr\cos\gamma)^2/k_e^2$$

$$r_p = p/(1 + \sqrt{1 - p/a})$$

$$H_p = r_p - r_e$$

$$H_a = ap/r_p - r_e$$

Integrate

Let X represent W q r ω_w U_x U_y U_z W_x W_y or W_z

For all Steps $\Delta x = \dot{x}\,\Delta t_I$

Step 1

$$x^1 = x + \Delta x/2$$

$$\Delta = \Delta x$$

$$t = t + \Delta t_I/2$$

Step 2

$$x^1 = x + (1 - 1/\sqrt{2})\,(\Delta x - \Delta)$$

$$\Delta = (2 - \sqrt{2})\Delta x + (-2 + 3/\sqrt{2})\,\Delta$$

Step 3

$$x^1 = x + (1 + 1/\sqrt{2})\,(\Delta x - \Delta)$$

$$\Delta = (2 + \sqrt{2})\,\Delta x + (-2 - 3/\sqrt{2})\Delta$$

$$t = t + \Delta t_I/2$$

Step 4

$$x^1 = x + \Delta x/6 - \Delta/3$$

After each step, $\dot{x}$ is recalculated for the next step.

APPENDIX III

COMPUTER PROGRAM

(SOURCE DECK LISTING)

FORTRAN IV - IBM 7040

```
 ISN       SOURCE STATEMENT

      C
      C        HARP  TRAJECTORY  PROGRAM  -  R. M. MCKEE
      C
      C          INPUT
      C
   1        DIMENSION AZS(10),C(10),DTB(10),ELS(10),EA(10),FR(10),
           1 I1(10),I2(10),Q(10),QT(10),SA(10),SI(10),WT(10),Y(10),YD(10)
   2   1000 CALL CLOCK (L)
   3        READ(5,1001)M,NS,(WT(NS+1)),(SA(NS+1)),DTI,DTP,
           1               VR,ELG,AZG,ONG,ATG,HT
  16   1001 FORMAT (2I5,F5.2,F5.3,3F5.0,4F5.2,F5.0)
  17        IF (NS.EQ.0) GO TO 2
  22        READ (5,1002) (I1(N),I2(N),WT(N),SA(N),EA(N),DTB(N),QT(N),
           1                ELS(N),AZS(N),FR(N),SI(N),N=1,NS)
  27   1002 FORMAT (2I5,F5.1,2F5.3,5F5.2,F5.0)
  30      2 NP = 1
  31        WRITE (6,1003) M,NP
  32   1003 FORMAT (1H1,15X,41HHARP  TRAJECTORY  PROGRAM  -  R. M. MCKEE,
           1         35X,4HCASE,I5,5X,4HPAGE,I3)
  33        WRITE (6,1004)M,VR
  34   1004 FORMAT (1H-,10X, 11HCASE NUMBER,6X,I7,9X,15HMUZZLE VELOCITY,
           1        2X,F7.0)
  35        WRITE (6,1005)NS,ELG
  36   1005 FORMAT (1H ,10X,13HNO. OF STAGES,4X,I7,9X, 9HELEVATION,8X,F7.2)
  37        WRITE (6,1006)WT(NS+1),AZG
  40   1006 FORMAT (1H ,10X,15HPAYLOAD  WEIGHT,2X,F7.2,9X, 7HAZIMUTH,10X,F7.2)
  41        WRITE (6,1007)SA(NS+1),ONG
  42   1007 FORMAT (1H ,19X,4HAREA,4X,F7.3,9X,9HLONGITUDE,8X,F7.2)
  43        WRITE (6,1008)DTI,ATG
  44   1008 FORMAT (1H ,10X,  9HSTEP SIZE,8X,F7.0,9X,8HLATITUDE,9X,F7.2)
  45        WRITE (6,1009)DTP,HT
  46   1009 FORMAT (1H ,10X,15HOUTPUT INTERVAL,2X,F7.0,9X,6HHEIGHT,11X,F7.0)
  47        IF (NS.EQ.0)  GO TO 1
  52        WRITE (6,1010)(N,N=1,NS)
  57   1010 FORMAT (1H-,10X,12HSTAGE NUMBER,6X,6I15)
  60        WRITE (6,1011)(I1(N),N=1,NS)
  65   1011 FORMAT (1H0,10X,18HIGNITION INDICATOR,6I15)
  66        WRITE (6,1012)(I2(N),N=1,NS)
  73   1012 FORMAT (1H ,10X,16HFIRING INDICATOR,2X,6I15)
  74        WRITE (6,1013)(WT(N),N=1,NS)
 101   1013 FORMAT (1H ,10X,14HWEIGHT (TOTAL),4X,6F15.1)
 102        WRITE (6,1014)(SA(N),N=1,NS)
 107   1014 FORMAT (1H ,10X,4HAREA,14X,6F15.3)
 110        WRITE (6,1015)(EA(N),N=1,NS)
 115   1015 FORMAT (1H ,10X,12HEXHAUST AREA,6X,6F15.3)
 116        WRITE (6,1019)(DTB(N),N=1,NS)
 123   1019 FORMAT (1H ,10X,12HBURNING TIME,6X,6F15.2)
 124        WRITE (6,1016)(QT(N),N=1,NS)
 131   1016 FORMAT (1H ,10X,14HIGNITION VALUE,4X,6F15.2)
 132        WRITE (6,1017)(ELS(N),N=1,NS)
 137   1017 FORMAT (1H ,10X,16HFIRING ELEVATION,2X,6F15.2)
 140        WRITE (6,1018)(AZS(N),N=1,NS)
 145   1018 FORMAT (1H ,10X,14HFIRING AZIMUTH,4X,6F15.2)
 146        WRITE (6,1026) (FR(N),N=1,NS)
 153   1026 FORMAT (1H ,10X,13HMASS FRACTION,5X,6F15.2)
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
     ISN         SOURCE STATEMENT

     154         WRITE (6,1020)(SI (N),N=1,NS)
     161    1020 FORMAT (1H ,10X,16HSPECIFIC IMPULSE,2X,6F15.0)
     162       1 WRITE (6,1025)
     163    1025 FORMAT (1H-, 7X,4HTIME,6X,6HHEIGHT,7X,5HRANGE,4X,8HVELOCITY,
               1  4X,8HAIRSPEED,2X,10HPATH ANGLE,3X,9HELEVATION,5X,7HAZIMUTH,
               2  5X,7HWIND EL,5X,7HWIND AZ/1H )
     164         NL = 28
     165         IF (NS.EQ.0) NL = 13
           C
           C          INITIALIZE  RUN
           C
     170      10 GC = 1.407639E16
     171         GO = 32.1465
     172         RE = 2.092564E7
     173         SP = 7.31958  E -5
     174         F = 57.295780
     175         ONO = ONG/F
     176         ATO = ATG/F
     177         AZ  = AZG/F
     200         EL  = ELG/F
     201         N = 1
     202         CT = 0
     203         S4 = -1.
     204         IF (NS.EQ.0) S4 = 0
     207         DT = DTI
     210         DTD = 0
     211         TH = 0
     212         YD(1) = 0
     213         T  = 0
     214         TO = 0
     215         TI = 0
     216         TB = 0
     217         Y(1) = WT(1)
     220         SLA = SIN(ATO)
     221         CLA = COS(ATO)
     222         SAZ = SIN(AZ)
     223         CAZ = COS(AZ)
     224         SEL = SIN(EL)
     225         CEL = COS(EL)
     226         Y(2) = VR*SEL
     227         Y(3) = RE+HT
     230         Y(4) = SQRT((VR*CEL*SAZ+SP*Y(3)*CLA)**2+(VR*CEL*CAZ)**2)/ Y(3)
     231         Y(5) =  CLA
     232         Y(6) =  0
     233         Y(7)=  SLA
     234         Y(8) = -(VR*CEL*SAZ+SP*Y(3)*CLA)*SLA/(Y(3)*Y(4))
     235         Y(9) = -VR*CEL*CAZ/(Y(3)*Y(4))
     236         Y(10) = (VR*CEL*SAZ+SP*Y(3)*CLA)*CLA/(Y(3)*Y(4))
           C
           C          INITIALIZE  STEP
           C
     237      20 VX = Y(7)*Y(9)-Y(6)*Y(10)
     240         VY = Y(5)*Y(10)-Y(7)*Y(8)
     241         VZ = Y(6)*Y(8)-Y(5)*Y(9)
     242         VRU = Y(2)
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
     ISN       SOURCE STATEMENT

     243          VRV = Y(3)*(Y(4)-SP*Y(10))
     244          VRW = Y(3)*SP*VZ
     245          VR =  SQRT(VRU**2+VRV**2+VRW**2)
     246          HT = Y(3)-RE
     247          V  = SQRT(Y(2)**2+(Y(3)*Y(4))**2)
     250          IF (T.NE.TB) PAP = PA
     253          PA = ATAN2(Y(2),Y(3)*Y(4))
     254          S3  = 2.
     255          IF (T.EQ.0.) S3 = 1.
     260          IF(HT.LT.0.)S4=0.
     263          DTD = DT
     264          DT = DTI
     265          IF (S4.EQ.0.)GO TO 27
     270          IF (S4.EQ.1.) GO TO 24
     273          IF (I1(N).EQ.1.AND.T.EQ.TB) GO TO 30
     276          IF (I1(N).EQ.1)     DTD = (QT(N)/F-PA)*DTD/(PA-PAP)
     301          IF (I1(N).EQ.0)     DTD = (1000.*QT(N)-HT)/Y(2)
     304          IF (I1(N).EQ.2  )  DTD = QT(N)-T+TB
     307          IF (I1(N).EQ.3) GO TO 21
     312          IF (ABS(DTD).GE.DTI) GO TO 30
     315          IF (ABS(DTD).LT..01) GO TO 22
     320          CT = CT+1.
     321          IF (CT.GT.20.)   GO TO 29
     324          DT = DTD
     325          GO TO 30
     326       22 CT = 0
     327          IF (T.GE.TB) GO TO 21
     332          NN = N-1
     333          WRITE (6,1107) N,NN
     334     1107 FORMAT (1HO,24HATTEMPTED IGNITION STAGE,I2,1X,
                1      20HBEFORE BURNOUT STAGE,I2)
     335          GO TO 70
     336       21 IF (I2(N).EQ.1.OR.I2(N).EQ.3) GO TO 23
     341          IF (I2(N).EQ.2  )  GO TO 25
     344          DU = Y(2)/V
     345          DV = Y(3)*Y(4)/V
     346          DW = 0
     347          GO TO 26
     350       25 EL  = ELS(N)/F
     351          AZ  = AZS(N)/F
     352          CEL = COS(EL)
     353          SEL = SIN(EL)
     354          CAZ = COS(AZ)
     355          SAZ = SIN(AZ)
     356          DU  = SEL
     357          DV  = CEL*CAZ
     360          DW  = -CEL*SAZ
     361       26 DX = DU*Y(5)+DV*VX+DW*Y(8)
     362          DY = DU*Y(6)+DV*VY+DW*Y(9)
     363          DZ = DU*Y(7)+DV*VZ+DW*Y(10)
     364       23 S4 = 1.
     365          S3 = 1.
     366          YD(1) =-FR(N)*(WT(N)-WT(N+1))/DTB(N)
     367          THO =-SI(N)*YD(1)
     370          TI = T
```

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HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN         SOURCE STATEMENT

      371         DT = DTB(N)/4.
      372         IF (DT.GT.DTI)  DT = DTI
      375         OND= F*(ATAN2(Y(6),Y(5))-SP*T+ONO)
      376         ATD= F*(ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2)))
      377         WRITE (6,1101)N,OND,ATD
      400    1101 FORMAT (1H ,5HSTAGE,I2,1X,20HIGNITION - LONGITUDE,F9.3,
                 1  9H LATITUDE,F9.3)
      401         NL = NL +1
      402         GO TO 30
      403      24 IF ((T+DT).GE.(TI+DTB(N)))  DT = TI+DTB(N)-T
      406         IF (DT.LT..01) S3 = 0
      411         GO TO 30
      412      27 IF (Y(2).GE.0.) GO TO 102
      415         DTD = -HT/Y(2)
      416         IF (ABS(DTD).GT.DTI)  GO TO 30
      421         IF (ABS(DTD).LT..01) GO TO 28
      424         CT = CT+ 1.
      425         IF (CT.GT.20.) GO TO 29
      430         DT = DTD
      431         GO TO 30
      432      28 OND= F*(ATAN2(Y(6),Y(5))-SP*T+ONO)
      433         ATD= F*(ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2)))
      434         WRITE (6,1103) OND,ATD
      435    1103 FORMAT (1H ,18HIMPACT - LONGITUDE,F9.3,9H LATITUDE,F9.3)
      436         NL = NL+1
      437         S3 = -1.
      440         GO TO 30
      441     102 IF (T.EQ.TB) GO TO 30
      444         DTD = -Y(2)/YD(2)
      445         IF (ABS(DTD).GT.DTI) GO TO 30
      450         IF (ABS(DTD).LT..01) GO TO 103
      453         CT = CT+1.
      454         IF (CT.GT.20.) GO TO 29
      457         DT = DTD
      460         GO TO 30
      461     103 CT = 0
      462         WRITE (6,1109)
      463    1109 FORMAT (1H ,6HAPOGEE)
      464         NL = NL+1
      465         S3 = 1.
      466         GO TO 30
      467      29 WRITE (6,1104)
      470    1104 FORMAT (1HO,17HITERATION FAILURE)
      471         GO TO 70
            C
            C          CALCULATE VARIABLES
            C
      472      30 DO 65 J = 1,4
      473         IF (NL.LT.55) GO TO 31
      476         NP = NP+1
      477         WRITE (6,1003) M,NP
      500         WRITE (6,1025)
      501         NL = 5
      502      31 VX = Y(7)*Y(9)-Y(6)*Y(10)
      503         VY = Y(5)*Y(10)-Y(7)*Y(8)
```

```
ISN        SOURCE STATEMENT

           VZ = Y(6)*Y(8)-Y(5)*Y(9)
           VRU = Y(2)
           VRV = Y(3)*(Y(4)-SP*Y(10))
           VRW = Y(3)*SP*VZ
           VR =  SQRT(VRU**2+VRV**2+VRW**2)
           HT = Y(3)-RE
           IF (HT.LT.1E6) GO TO 32
           AP = 0.
           DR = 0.
           GO TO 40
        32 AP = 2116.22*EXP(-4.42E-5*HT)
      C
      C          CALCULATE  THRUST
      C
        40 IF (S4.NE. 1.)  GO TO 50
           TH = THO-EA(N)*AP
           IF (I2(N).EQ.1 )   GO TO 41
           IF (I2(N).EQ.3) GO TO 43
           DU = DX*Y(5)+DY*Y(6)+DZ*Y(7)
           DV = DX*VX  +DY*VY  +DZ*VZ
           DW = DX*Y(8)+DY*Y(9)+DZ*Y(10)
           GO TO 50
        41 DU = VRU/VR
           DV = VRV/VR
           DW = VRW/VR
           GO TO 50
        43 DU = Y(2)/V
           DV = Y(3)*Y(4)/V
           DW = 0
      C
      C          CALCULATE  DRAG  AND  NORMAL  FORCE
      C
        50 IF (HT.LT.36200.)                      SS = 1116.45 - .0040986*HT
           IF (HT.GE.36200..AND.HT.LT.65800.)  SS =  968.08
           IF (HT.GE.65800..AND.HT.LT.105000.) SS =   924.28 + .0006658*HT
           IF (HT.GE.105000..AND.HT.LT.155500.)SS =  811.54 + .0017394*HT
           IF (HT.GE.155500..AND.HT.LT.172000.)SS =  1082.02
           IF (HT.GE.172000..AND.HT.LT.202000.)SS =  1291.40 - .0012173*HT
           IF (HT.GE.202000..AND.HT.LT.262500.)SS =  1584.72- .0026694*HT
           IF (HT.GE.262500.)                     SS =  884.0
           EM = VR/SS
           IF (EM.LT.1.) CD = .35
           IF (EM.GE.1.) CD = .18+EXP(-EM/2.)
           QS=.7*AP*EM**2*SA(N)
           DR = CD*QS
           IF (S4.NE.1..OR.I2(N).EQ.1) GO TO 51
           CNA = 2.+12.*EXP(-EM/4.)-10.*EXP(-EM)
           QN = CNA*QS/VR
           FNU = QN*(DU*(DV*VRV+DW*VRW)-VRU*(DV**2+DW**2))
           FNV = QN*(DV*(DU*VRU+DW*VRW)-VRV*(DU**2+DW**2))
           FNW = QN*(DW*(DU*VRU+DV*VRV)-VRW*(DU**2+DV**2))
           FN = SQRT(FNU**2+FNV**2+FNW**2)
           IF  (FN/Y(1).LT.20.) GO TO 55
           WRITE (6,1115)
      1115 FORMAT (33H NORMAL ACCELERATION EXCEEDS 20 G)
```

```
     ISN        SOURCE STATEMENT

     625           GO TO 70
     626        51 FNU = 0.
     627           FNV = 0.
     630           FNW = 0.
           C
           C           CALCULATE  RATES
           C
     631        55 YD(2) = Y(3)*Y(4)**2+(GO/Y(1))*(TH*DU-DR*VRU/VR+FNU)-GC/Y(3)**2
     632           YD(3) = Y(2)
     633           YD(4) = (-2.*Y(2)*Y(4)+(GO/Y(1))*(TH*DV-DR*VRV/VR+FNV))/Y(3)
     634           WU    = (TH*DW-DR*VRW/VR+FNW)*GO/(Y(1)*Y(3)*Y(4))
     635           YD(5) = VX*Y(4)
     636           YD(6) = VY*Y(4)
     637           YD(7) = VZ*Y(4)
     640           YD(8) = -VX*WU
     641           YD(9) = -VY*WU
     642           YD(10) = -VZ*WU
           C
           C           OUTPUT
           C
     643        80 IF (S3.EQ.2..AND.T.LT.(TO+DTP).OR.J.NE.1) GO TO 60
     646           TO = T
     647           ON = ATAN2(Y(6),Y(5))-SP*T+ONO
     650           AT = ATAN2(Y(7),SQRT(Y(5)**2+Y(6)**2))
     651           IF (T.EQ.0.) GO TO 83
     654           AZ = ATAN2(SIN(ON-ONO)*COS(AT-ATO),SIN(AT-ATO))
     655           RA = RE*ATAN2(ABS(SIN(ON-ONO)),ABS(SIN(AZ))*COS(ON-ONO))
     656           GO TO 84
     657        83 RA = 0
     660        84 RAN = RA/6076.104
     661           HTN = HT/6076.104
     662           WELD = F*ATAN2(VRU,SQRT(VRV**2+VRW**2))
     663           WAZD = F*ATAN2(-VRW,VRV)
     664           IF (S4.EQ.1.) GO TO 81
     667           ELD = WELD
     670           AZD = WAZD
     671           GO TO 82
     672        81 ELD = F*ATAN2(DU,SQRT(DV**2+DW**2))
     673           AZD = F*ATAN2(-DW,DV)
     674        82 PAD = F*PA
     675           WRITE (6,1105)T,HTN,RAN,V,VR,PAD,ELD,AZD,WELD,WAZD
     676      1105 FORMAT (3F12.2,2F12.0,5F12.2)
     677           NL = NL +1
     700           IF (S3.NE.0.) GO TO 60
     703           OND = F*ON
     704           ATD = F*AT
     705           WRITE (6,1102)N,OND,ATD
     706      1102 FORMAT (1H ,5HSTAGE,I2,20H BURNOUT - LONGITUDE,F9.3,
                 1  9H LATITUDE,F9.3)
     707           NL = NL+1
     710           N=N+1
     711           S4 = -1.
     712           IF (N.GT.NS)    S4 = 0
     715            YD(1) = 0
     716           TH = 0
```

```
HARP TRAJ. PROG. R. M. MCKEE                    FORTRAN SOURCE LIST RMCKEE
      ISN          SOURCE STATEMENT

      717          TB = T
      720          DT = DTI
      721          Y(1) = WT(N)
      722          IF (HT.LT.2.5E5) GO TO 20
      725          AX = 1./(2./Y(3)-V**2/GC)
      726          SL = (V*Y(3)*COS(PA))**2/GC
      727          IF (SL/AX.LT.1.) GO TO 85
      732          WRITE (6,1110)
      733     1110 FORMAT (17H HYPERBOLIC ORBIT)
      734          NL = NL+1
      735          GO TO 86
      736       85 RP = SL/(1.+SQRT(1.-SL/AX))
      737          HP = (RP-RE)/6076.104
      740          HA = (AX*SL/RP-RE)/6076.104
      741          WRITE (6,1106)HP,HA
      742     1106 FORMAT (21X,7HPERIGEE,F9.2,3X,6HAPOGEE,F9.2)
      743          NL = NL+1
      744          IF (HP.LT.50.)   GO TO 20
      747       86 IF (S4.EQ.0.) GO TO 70
      752          GO TO 20
             C
             C           INTEGRATE
             C
      753       60 IF (S3.EQ.-1.) GO TO 70
      756          DO 65 I=1,10
      757          C(I) = YD(I)*DT
      760          GO TO (61,62,63,64),J
      761       61 Y(I) = Y(I)+C(I)/2.
      762          Q(I) = C(I)
      763          IF (I.EQ.1) T = T+DT/2.
      766          GO TO 65
      767       62 Y(I) = Y(I)+.29289322*(C(I)-Q(I))
      770          Q(I) = .58578644*C(I)+.12132034*Q(I)
      771          GO TO 65
      772       63 Y(I) = Y(I)+1.7071068*(C(I)-Q(I))
      773          Q(I) = 3.4142136*C(I)-4.1213203*Q(I)
      774          IF (I.EQ.1) T = T+DT/2.
      777          GO TO 65
     1000       64 Y(I) = Y(I)+C(I)/6.-Q(I)/3.
     1001       65 CONTINUE
     1004          GO TO 20
             C
             C           DIAGNOSTICS
             C
     1005       70 AVU = SQRT(Y(5)**2+Y(6)**2+Y(7)**2)-1.
     1006          AVW = SQRT(Y(8)**2+Y(9)**2+Y(10)**2)-1.
     1007          SPWU = Y(8)*Y(5)+Y(9)*Y(6)+Y(10)*Y(7)
     1010          WRITE (6,1108)AVU,AVW,SPWU
     1011     1108 FORMAT (1H0,18HACCURACY CHECK - U,F12.8,3H  W,F12.8,5H  U.W,F12.8)
     1012          CALL OVERFL (K)
     1013          GO TO (71,73,72),K
     1014       71 WRITE (6,1111)
     1015     1111 FORMAT (9H OVERFLOW)
     1016          GO TO 73
     1017       72 WRITE (6,1112)
```

HARP TRAJ. PROG. R. M. MCKEE FORTRAN SOURCE LIST RMCKEE

```
 ISN        SOURCE STATEMENT

1020   1112 FORMAT (10H UNDERFLOW)
1021     73 CALL DVCHK (K)
1022        IF (K.EQ.2) GO TO 74
1025        WRITE (6,1113)
1026   1113 FORMAT (13H DIVIDE CHECK)
1027     74 LO = L
1030        CALL CLOCK (L)
1031         DL = L-LO
1032        DLS = DL/60.
1033        WRITE (6,1114)DLS
1034   1114 FORMAT (15H COMPUTING TIME,F8.2,8H SECONDS)
1035        GO TO 1000
          C
1036        END
```

PLUS ROUTINE FPT (UNLIMITED FLOATING POINT TRAPS)

APPENDIX IV

INPUTS

A complete list of inputs is given on the next page, with symbols, definitions and units. They are divided into two groups corresponding to the two types of data cards, and the order is the same as that on the data cards. The indicators and ignition value are defined as follows:-

Ignition Indicator	Meaning	Ignition Value
	Ignition at given	
0	- Height	Height (ft)/1000
1	- Flight Path Angle	Angle (deg)
2	- Time after Previous Burnout	Interval (sec)
Other	- Burnout of Previous Stage	

Firing Indicator	Meaning
	Direction of roll axis during firing
0	- fixed, tangent to absolute flight path angle at ignition
1	- parallel to relative velocity (aero-stab.)
2	- fixed, given elevation and azimuth at ignition
3	- parallel to absolute velocity
Other	- as 0

The layout of the data cards is shown on the page following the list of inputs. They are of two types, one for gun and general parameters, and one for each of the rocket stages.

All data fields are five columns wide. Decimal points need not be punched (and must not be in the first two fields). The assumed number of places to the right of the decimal is indicated. This format will be superseded by a decimal point if punched, but the formal of the input print-out will not then agree. These cards must be ordered according to the sequence of operation in the real system, for each case; i.e., a gun card followed by a card for the first stage, a card for the second stage, etc. The number of stage cards must be the same as the number of stages quoted on the gun card. The sets of cards for successive cases follow one another.

The integration interval is ideally one which produces the least errors. As the interval is increased, the mathematical errors predominate; as it is decreased, round off errors take over. For most purposes, 5 seconds should be about right.

SYMBOL			
FORTRAN	MATH	DESCRIPTION	
M	Case	Case Number	
NS	n_s	Number of Stages	
WT(NS+1)	W_{ns+1}	Payload Weight	(lb)
SA(NS+1)	S_{ns+1}	Payload Area	(ft^2)
DTI	Δt_I	Step Size for Integration	(sec)
DTP	Δt_p	Nominal Output Interval	(sec)
VR	V_R	Muzzle Velocity	(ft/sec)
ELG	δ_G	Gun Elevation from Horizontal	(deg)

SYMBOL		DESCRIPTION	
FORTRAN	MATH		
AZG	β_g	Gun Azimuth from North	(deg)
ON	θ_g	Gun Longitude (Positive East)	(deg)
ATG	ϕ_g	Gun Latitude (Positive North)	(deg)
HT	H_g	Muzzle Height Above S.L.	(ft)
I1(N)	I_{1n}	Ignition Indicator	
I2(N)	I_{2n}	Thrust Direction Indicator	
WT(N)	W_n	Total Weight Before Ignition	(lb)
SA(N)	S_n	Cross Section Area	(ft^2)
EA(N)	A_n	Exhaust Area	(ft^2)
DTB(N)	Δt_{Bn}	Burning Time	(sec)
QT(N)	Q_n	Ignition Value	
ELS(N)	δ_{sn}	Thrust Elevation above Horizontal	(deg)
AZS(N)	β_{sn}	Thrust Azimuth from Track	(deg)
FR(N)	f_n	Mass Fraction	
SI(N)	I_n	Specific Impulse in Vacuum	(sec

INPUT DATA FORMAT AND DECK MAKE-UP

Column	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60
General Data Card	Case	n_s	W_{ns+1}	S_{ns+1}	Δt_I	Δt_p	V_R	δ_g^o	β_g^o	θ_g^o	ϕ_g^o	H_g
Decimal Places	0	0	2	3	0	0	0	2	2	2	2	0
Stage Data Card	I_1	I_2	W	S	A	Δt_B	Q	δ_s	β_s	f	I	
Decimal Places	0	0	1	3	3	2	2	2	2	2	0	

Column	1 - 6	8 - 10	11 - 13	14 - 15	16 - 21	18 - 45
Deck Make-Up A Control Cards - 1	$JOB	(JOB CODE)	t	n		(Identificati
2	$IBJOB				Nodeck	
3	$IBFTC	(DECK NAME)				
Source Deck						
FPT Deck						
Entry Card	$ENTRY				(Deck Name)	
Data Deck						
Final Card	$IBSYS					
Deck Make-Up B Control Cards - 1	$JOB	(JOB CODE)	t	n		(Identificati
2	$IBJOB				No source	
Object Deck						
FPT Deck						
Entry Card	$ENTRY				(Deck Name)	
Data Deck						
Final Card	$IBSYS					

APPENDIX V

DECK MAKE-UP

The order of the complete deck and the details of the control cards for use on the IBM 7040 at McGill are shown on the preceding page. The first is for use with a source deck (as listed in Appendix III). Normally, this would be used only when making program changes; when these have proven satisfactory, the second card should not be punched NODECK as shown; an object deck will then be produced. The second shows the deck set-up for normal operation, with the object deck. The FPT deck is an object deck of a special routine which allows unlimited floating point traps; the normal procedure is for the monitor to terminate execution when the recorded number of floating point traps reaches a given amount.

On the first card "t" is the estimated computing time in minutes including compilation, and "n" is the estimated number of pages of output. If either of these is exceeded, the job will be terminated. The computer time may be estimated as $3.8 + t_f/18 \Delta t_I$ (or $t_f/10 \Delta t_p$, whichever is larger) seconds computing time per case where t_f is the expected flight time in seconds, plus compiling time of 4 minutes if a source deck is used. The number of pages of output may be estimated as $1 + t_f/55 \Delta t_p$ per case rounded to the next higher integer, plus 8 for the listing if a source deck is used, plus 3 for bookkeeping.

APPENDIX VI

OUTPUTS

The print-out for each case begins with a complete listing of all inputs, followed by the column headings for the periodic records which are printed at intervals of Δt_p and at events (launch, ignition, burnout, apogee and impact).

TIME	....	time in seconds from launch
HEIGHT	....	above sea level in nautical miles
RANGE	....	at sea level (great circle) in nautical miles
VELOCITY	....	absolute velocity in ft/sec
AIRSPEED	....	velocity relative to rotating earth in ft/sec
PATH ANGLE	....	angle of elevation of the tangent to the absolute flight path above the local horizontal in degrees
ELEVATION	....	angle of elevation of the vehicle roll axis above the local horizontal, in degrees
AZIMUTH	....	angle from the horizontal velocity vector to the horizontal projection of the vehicle roll axis, measured clockwise as viewed from above, in degrees
WIND EL	....	as elevation, for the relative velocity in degrees
WIND AZ	....	as azimuth, for the relative velocity, in degrees

In addition, messages are produced at the appropriate times to indicate ignition, burnout, apogee and impact. Except for apogee, these are accompanied by latitude (in degrees North) and longitude (in degrees East).

When burnout occurs at a height greater than 250,000 ft., perigee and apogee heights are calculated and presented in nautical miles, unless the orbit is hyperbolic, which fact is indicated.

Various error conditions are indicated by appropriate messages. "Attempted ignition (of a) stage before burnout (of the previous) stage" is caused by incompatible inputs. "Iteration failure", "overflow" or "divide check" require detailed examination of the program. "Underflow" is not significant. "Accuracy check" shows the errors in the unit vectors on the U and W axes and their scalar product after completion of the case. These should be of the order of the round off error, i.e. of the order of $t_f/10^8 \Delta t_I$. An arbitrary limit of 20G was placed on the normal acceleration, and exceeding this figure results in termination of the calculation with the appropriate message.

A sample output is shown on the following pages.

HARP TRAJECTORY PROGRAM - R. M. MCKEE   CASE -0   PAGE 1

CASE NUMBER	-0	MUZZLE VELOCITY	4500.
NO. OF STAGES	3	ELEVATION	38.00
PAYLOAD WEIGHT	50.00	AZIMUTH	118.23
AP A	1.478	LONGITUDE	-59.48
STEP SIZE	5.	LATITUDE	13.07
OUTPUT INTERVAL	5.	HEIGHT	150.

STAGE NUMBER	1	2	3
IGNITION INDICATOR	0	2	1
FIRING INDICATOR	1	2	2
WEIGHT (TOTAL)	2000.0	600.0	180.0
AREA	1.478	1.478	1.478
EXHAUST AREA	0.785	0.785	0.785
BURNING TIME	20.00	20.00	20.00
IGNITION VALUE	25.00	10.00	0.38
FIRING ELEVATION	-0.	3.00	0.
FIRING AZIMUTH	-0.	0.	0.
MASS FRACTION	0.80	0.80	0.80
SPECIFIC IMPULSE	300.	300.	300.

TIME	HEIGHT	RANGE	VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	AZIMUTH	WIND EL	WIND AZ
0.	0.02	0.	5639.	4500.	29.43	38.00	8.26	38.00	8.26
5.00	2.06	2.74	4945.	3772.	26.84	36.29	9.22	36.29	9.22
10.00	3.75	5.16	4545.	3340.	24.44	34.25	9.86	34.25	9.86
STAGE 1 IGNITION - LONGITUDE -59.396 LATITUDE 13.026									
11.21	4.11	5.72	4472.	3260.	23.87	33.71	9.98	33.71	9.98
16.21	5.79	8.39	5558.	4350.	24.32	31.74	8.05	31.74	8.05
21.21	7.91	11.99	6916.	5710.	24.59	30.26	6.49	30.26	6.49
26.21	10.58	16.76	8678.	7474.	24.81	29.16	5.19	29.16	5.19
31.21	13.98	23.07	11058.	9857.	25.05	28.36	4.09	28.36	4.09
STAGE 1 BURNOUT - LONGITUDE -59.140 LATITUDE 12.891									
36.21	17.74	30.24	10834.	9625.	24.38	27.69	4.16	27.69	4.16
STAGE 2 IGNITION - LONGITUDE -58.931 LATITUDE 12.780									
41.21	21.35	37.32	10716.	9499.	23.73	3.00	-0.	27.00	4.20
46.21	24.81	44.95	11967.	10707.	20.09	3.14	0.12	22.58	3.58
51.21	28.15	53.86	13539.	12250.	17.27	3.31	0.15	19.15	3.07
56.21	31.44	64.31	15515.	14205.	14.94	3.50	0.15	16.35	2.61
61.21	34.75	76.68	18089.	16763.	12.95	3.72	0.14	14.00	2.19
STAGE 2 BURNOUT - LONGITUDE -58.346 LATITUDE 12.486									
66.21	38.05	90.17	18049.	16719.	12.70	13.73	2.20	13.73	2.20
71.21	41.28	103.62	18012.	16680.	12.45	13.47	2.21	13.47	2.21
76.21	44.44	117.04	17977.	16643.	12.20	13.20	2.22	13.20	2.22
81.21	47.53	130.43	17944.	16607.	11.95	12.93	2.23	12.93	2.23
86.21	50.56	143.79	17911.	16572.	11.70	12.66	2.24	12.66	2.24
91.21	53.51	157.12	17880.	16538.	11.45	12.39	2.25	12.39	2.25
96.21	56.40	170.42	17849.	16505.	11.19	12.12	2.26	12.12	2.26
101.21	59.21	183.69	17820.	16473.	10.94	11.85	2.27	11.85	2.27
106.21	61.96	196.94	17790.	16442.	10.68	11.57	2.28	11.57	2.28

HARP TRAJECTORY PROGRAM - R. H. MCKEE CASE -0 PAGE 2

TIME	HEIGHT	RANGE	VELOCITY	AIRSPEED	PATH ANGLE	ELEVATION	AZIMUTH	WIND EL	WIND AZ
111.21	64.64	210.16	17762.	16411.	10.43	11.30	2.28	11.30	2.28
116.21	67.25	223.35	17734.	16382.	10.17	11.02	2.29	11.02	2.29
121.21	69.80	236.52	17707.	16353.	9.91	10.74	2.30	10.74	2.30
126.21	72.27	249.66	17681.	16324.	9.65	10.47	2.31	10.47	2.31
131.21	74.68	262.78	17656.	16297.	9.40	10.19	2.32	10.19	2.32
136.21	77.01	275.88	17631.	16270.	9.14	9.91	2.32	9.91	2.32
141.21	79.28	288.95	17607.	16244.	8.87	9.63	2.33	9.63	2.33
146.21	81.48	302.01	17584.	16219.	8.61	9.34	2.34	9.34	2.34
151.21	83.62	315.04	17561.	16195.	8.35	9.06	2.35	9.06	2.35
156.21	85.68	328.04	17539.	16171.	8.09	8.78	2.35	8.78	2.35
161.21	87.68	341.03	17518.	16148.	7.83	8.49	2.36	8.49	2.36
166.21	89.61	354.00	17497.	16126.	7.56	8.21	2.37	8.21	2.37
171.21	91.47	366.95	17478.	16105.	7.30	7.92	2.37	7.92	2.37
176.21	93.26	379.88	17459.	16084.	7.03	7.64	2.38	7.64	2.38
181.21	94.98	392.79	17440.	16065.	6.77	7.35	2.39	7.35	2.39
186.21	96.64	405.68	17423.	16046.	6.50	7.06	2.39	7.06	2.39
191.21	98.23	418.56	17406.	16027.	6.23	6.77	2.40	6.77	2.40
196.21	99.75	431.42	17390.	16010.	5.97	6.48	2.41	6.48	2.41
201.21	101.20	444.27	17374.	15993.	5.70	6.19	2.41	6.19	2.41
206.21	102.59	457.10	17360.	15978.	5.43	5.90	2.42	5.90	2.42
211.21	103.91	469.91	17346.	15962.	5.16	5.61	2.42	5.61	2.42
216.21	105.16	482.71	17332.	15948.	4.89	5.32	2.43	5.32	2.43
221.21	106.34	495.50	17320.	15935.	4.62	5.03	2.44	5.03	2.44
226.21	107.46	508.27	17308.	15922.	4.35	4.73	2.44	4.73	2.44
231.21	108.50	521.03	17297.	15910.	4.08	4.44	2.45	4.44	2.45
236.21	109.48	533.78	17286.	15899.	3.81	4.15	2.45	4.15	2.45
241.21	110.40	546.52	17277.	15888.	3.54	3.85	2.46	3.85	2.46
246.21	111.24	559.24	17268.	15878.	3.27	3.56	2.46	3.56	2.46
251.21	112.02	571.96	17260.	15870.	3.00	3.26	2.47	3.26	2.47
256.21	112.73	584.66	17252.	15861.	2.73	2.97	2.47	2.97	2.47
261.21	113.37	597.36	17245.	15854.	2.46	2.67	2.47	2.67	2.47
266.21	113.95	610.05	17239.	15847.	2.19	2.38	2.48	2.38	2.48
271.21	114.45	622.73	17234.	15842.	1.91	2.08	2.48	2.08	2.48
276.21	114.89	635.40	17229.	15837.	1.64	1.79	2.49	1.79	2.49
281.21	115.27	648.06	17225.	15832.	1.37	1.49	2.49	1.49	2.49
286.21	115.57	660.72	17222.	15829.	1.10	1.19	2.49	1.19	2.49
291.21	115.81	673.37	17219.	15826.	0.85	0.90	2.50	0.90	2.50
296.21	115.98	686.02	17218.	15824.	0.55	0.60	2.50	0.60	2.50
STAGE 3 IGNITION - LONGITUDE		-49.208	LATITUDE	7.768					
299.39	116.05	694.05	17217.	15823.	0.38	0.00	0.00	0.41	2.50
304.39	116.12	707.27	18721.	17326.	0.13	0.24	0.00	0.14	2.29
309.39	116.13	721.80	20504.	19108.	-0.02	0.50	0.00	-0.03	2.08
314.39	116.11	737.89	22694.	21296.	-0.07	0.78	0.00	-0.07	1.87
319.39	116.10	755.96	25531.	24133.	0.01	1.10	-0.00	0.01	1.65

STAGE 3 BURNOUT - LONGITUDE -48.295 LATITUDE 7.282
PERIGEE 116.04 APOGEE 128.04

ACCURACY CHECK - U -0.00000126 W -0.00000175 U.W 0.00000001
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13. ABSTRACT

In order to assess the potential of multi stage gun-launched rockets, a study of the vehicle and trajectory parameters was undertaken. A digital computer program for trajectories was written and was used in an experimental manner to approach optimum performance within various sets of restricting assumptions. The approach was found to be effective and a useful orbital potential was demonstrated with reasonable design parameters. (U)

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